Emergency Fund Levels: Is Household Behavior Rational?

Y. Regina Chang, University of Missouri-Columbia¹ Sherman Hanna, The Ohio State University² Jessie X. Fan, University of Utah³

Empirical studies have found that most households do not have recommended levels of emergency fund reserve to cover expenses in case of income disruption. A three period model of optimal consumption is presented. The theoretical model suggests that many consumers without recommended levels of liquid asset holding may be acting rationally. The model is tested empirically with the 1983-1986 panels of the Surveys of Consumer Finances. Empirical findings support the model in that households who could have expected to have decreases in future real income were significantly more likely to hold adequate emergency fund reserves than those who could have expected to have no decline in real income.

KEY WORDS: economic model, emergency funds, financial ratios, liquidity, Survey of Consumer Finance

Introduction

"To be prepared for the unexpected, people should have a reserve fund - equal to at least three to six months' living expenses - invested in a combination of low-risk money funds and CDS, plus smaller amounts of riskier but higher-yielding investments, such as short- and medium-term bond funds." (Asinof, 1992).

Emergency funds are usually defined as liquid assets because they are easily and quickly converted to cash for the needs of unexpected expenses (Johnson and Widdows, 1985; Prather, 1990). Recommendations of a level for an adequate fund to meet emergencies range from 2 to 6 months of expenses in liquid form (Johnson & Widdows, 1985; Prather, 1990). A survey of 156 financial planners and educators found that the average recommendation was that liquid assets amount to about three months of living expenses (Greninger, Hampton, Kitt & Achacoso, 1996). Garman and Forgue (1997, 77-78) suggest that the appropriate amount for a particular family depends on the family situation and job. "A smaller amount may be sufficient if you have adequate loss of income protection through an employee fringe benefit program or a union, are employed in a job that is definitely not subject to layoffs, or have an employed spouse." (Garman & Forgue, 1997, 77-78).

Previous empirical studies have found that most U.S. households do not meet the recommended standards. This paper develops an original three period model of consumption for determining optimal saving in order to provide insight into rational levels of emergency reserves. An empirical test of the model using the 1983-1986 panels of the Surveys of Consumer Finance (SCF) is presented. Implications for consumer education and for further research are discussed.

The Literature

Empirical Studies

Johnson and Widdows (1985) defined emergency funds as financial holdings which are made available to cover spending, without altering the current household standard of living, in the event of income disruption. The Johnson and Widdows (1985) study uses three measures of

¹Y. Regina Chang, Assistant Professor, Department of Consumer and Family Economics, University of Missouri-Columbia, 239 Stanley Hall, Columbia, MO 65211. Phone: (314) 882-9343. Email: cfechang@showme.missouri.edu

²Sherman Hanna, Professor, Consumer and Texile Sciences Department, The Ohio State University, 1787 Neil Ave., Columbus, OH 43210-1295. Phone: (614) 292-4584. FAX: (614) 292-7536. Email: hanna.1@osu.edu.

³Jessie X. Fan, Assistant Professor, Family and Consumer Studies, University of Utah, 228 Alfred Emery Building, Salt Lake City, UT 84112. Phone: (801) 581-4170. FAX: (801) 581-5156. Email: fan@fcs.utah.edu.

emergency fund holdings -- quick, intermediate and comprehensive -- which vary in their degree of liquidity of assets. Griffith (1985) proposed 16 ratios with various components of net worth to analyze a family's financial situation. Liquid assets were used in nine ratios, which provide insights into the adequacy of emergency fund holdings to cover expenses of unexpected financial crises.

Various studies have tried to determine what proportion of households meet recommended levels of emergency fund holdings (Chang & Huston, 1995; Chang ,1995; DeVaney, 1995; Hanna & Wang,1995; Hanna, Chang, Fan & Bae, 1993; Prather, 1990; Johnson & Widdows, 1985). Appendix 1 summarizes results found in these empirical studies. Despite differences in measurements and data used in the empirical analyses, these studies share a common conclusion in that a large proportion of households did not meet the three-month and six-month emergency fund guidelines

Previous studies also have tried to explore factors affecting the level of emergency fund a household holds. For instance, Lindqvist (1981), in a study of determinants of household savings in 429 Swedish families, found that income, family size and stage of life cycle were not significantly related to stocks of liquid assets, but that variables reflecting socio-psychological attributes of households, such as expectations and economic satisfaction, were significant.

Using data from the 1977 and 1983 Survey of Consumer Finances, Johnson and Widdows (1985) analyzed households' holdings of three types of emergency fund holding (quick emergency fund, intermediate emergency fund, and comprehensive emergency fund). The analysis revealed that the majority of families had insufficient funds to cover normal total household income for the average time a household could expect to be out of work, should that event occur. In 1983, using the broadest measure of emergency funds, only 19% of households had liquid asset holdings sufficient to cover six months of pretax income. The median level of liquid asset holding using the broad measure was seven percent of pretax income. In 1982-83, the mean level of consumer spending in U.S. households was 83% of the mean level of pretax income (USBLS, 1986), so the median level as a percent of annual spending was probably somewhat higher than seven percent. Johnson and Widdows (1985)

also showed that families, on the average, were less prepared for financial emergencies in 1983 than in 1977, indicating a macroeconomic effect on emergency fund holdingss. Moreover, the empirical results suggested a strong and positive relationship between income and emergency fund holdings, although other variables were not controlled. In a cross-tabulation of emergency fund levels by stages of the life cycle, the findings showed that in each case of emergency fund measure, households in the "young family stage" of the life cycle had the greatest concentration of emergency funds in the "less than two months' reserve" category. The concentration of families moved from the lower levels of emergency funds reserve to higher levels as families move through the life cycle (Johnson & Widdows, 1985). One limitation of the Johnson and Widdows study is that income rather than spending was used to evaluate the adequacy of liquid savings. This limitation is inherent in the U.S. datasets available, as the Survey of Consumer Finances contains the best balance sheet information but little information about spending, while the Survey of Consumer Expenditures contains the best expenditure information, but only limited information about household balance sheets.

Using the ratios proposed by Griffith (1985), Prather (1990) analyzed the 1983 Survey of Consumer Finance data and found that only 29% of households had liquid assets sufficient to cover six months of estimated spending. Prather found that income and age were significantly related to ratios of liquid assets to monthly expenses, total debt, non-mortgage debt, net worth, and one year's debt payment. These results are consistent with the Johnson and Widdows findings (1985).

Hanna, Chang, Fan, and Bae (1993) analyzed 1990 Bureau of Labor Statistics Consumer Expenditure interview data, for households with four quarters of data, and found that the proportions meeting the emergency fund guidelines were approximately the same using pretax income, aftertax income, or expenditures. Therefore, it is possible that analysis based on survey data lacking expenditure data may give reasonable results.

Theoretical Literature

There has been extensive discussion in the literature of theoretical models of optimal savings and consumption behavior under uncertainty either in the context of infinite time horizon or in two-period or multiperiod intertemporal models (e.g., Leland, 1968; Levhari & Srinivasan, 1969; Sandmo, 1970; Mirman, 1971; Dreze & Modigliani, 1972; Hey, 1979; Sibley, 1975; Salyer, 1988). In general, these authors analyzed one or two variables at a time while assuming a value for each of the other parameters. Holding liquid assets for a emergency fund can be seen as similar to buying insurance, with the loss on the potential rate of return for liquid assets compared to other assets being similar to the load on an insurance policy. Therefore, not holding the recommended level of liquid assets for an emergency fund can be seen as similar to having a high deductible on an insurance policy (c.f., Hanna, 1989).

In the discussion of income uncertainty and saving behavior, it is assumed that the consumer's belief about the value of future income can be summarized in a subjective probability density function; on the basis of this the consumer maximizes expected utility of consumption. Leland (1968) used a two-period model of consumption to demonstrate the effect of uncertainty on saving and concludes that with an additive utility function and the assumption of decreasing absolute risk aversion, the precautionary demand for savings is a positive function of uncertainty. Sandmo (1970) discussed the effects of increased riskiness of future income on present consumption in a two-period model and proved that an increase in uncertainty about future income decreased consumption (or increased savings). Sibley (1975) extended the two-period results of the effects on optimal savings of increased riskiness in the future income due to Leland (1968) to a multiperiod case. He suggested that increased wage uncertainty raise or lower savings depending upon whether the third derivative of the utility function is positive or negative. Since the plausible requirement that the consumer's utility function displays decreasing absolute risk aversion implies a positive third derivative, this establishes a presumption that optimal savings increases with wage uncertainty (Sibley, 1975). For the case of a constant (but negative) elasticity utility function, Levhari and Srinivasan (1969) also showed a positive relationship between optimal savings and uncertainty. The studies discussed above, however, only focused on the effects of subjective probability density function as a projection of uncertain future income on saving behavior. No study has been done in

incorporating possible factors such as level of risk aversion, interest rate, income, and income growth rate into the model to demonstrate the effects of these uncertainties on optimal saving behavior.

Factors affecting optimal saving include the expected growth rate of real income, the variance of future income, the consumer's utility function (e.g., the parameter of risk aversion), the real interest rate and the consumer's personal discount rate. For an exposition of a two period model, see Chang, Fan and Hanna (1992) or Fan, Chang and Hanna (1993). Other factors may be important, but are difficult to incorporate into a rigorous theoretical model. For instance, because of the existence of meanstested social insurance programs, it might be rational for low income households to hold relatively low levels of emergency funds (Hubbard, Skinner & Zeldes, 1995).

Theoretical Model

The present study includes factors which influence optimal saving decisions in a three period model of consumption. Kinsey and Lane (1978) suggested when consumption is accompanied by the use of consumer credit, utility maximization may be viewed in the global sense, thus a life cycle approach to the allocation of income, consumption, and savings (borrowing) is appropriate. While a multi-period model is very complicated and not feasible for this analysis, a threeperiod model can simulate the life cycle situation better than a two period model. A three-period model with uncertainty for determining optimal savings facing consumers is presented and illustrated with numerical analysis. An empirical model is tested based on the theoretical model.

A Three-Period Model of Consumption^a

To begin, consider the following model: assume that the consumer attempts to maximize the expected value of utility for the three periods. The saving decision is based first period income, which is known with certainty, and expectations of second and third period income. The second and third period consumption will depend on how much the consumer saves in the first period and on the actual value of second and third period income. The optimal amount to save should depend on the expected income growth rate (which may be negative) and the probability that income growth occurs, and also on the

real interest rate. For simplicity, it is assumed that there are two states of the world in the second period -- real income either decreases or stays constant, and in the third period, income will keep the level of the second period, no matter whatever happened in the second period. (The analysis could allow for other scenarios, but the discussion is limited to this scenario because it is the most plausible scenario for saving to be rational.) There are other motivations for holding liquid assets than as a buffer stock for income decreases, such as preparing for accidents or illnesses, or saving to purchase durable goods. This paper will ignore those motivations for holding liquid assets. For many households, private or public insurance may be relied upon for medical costs, and credit may be used to purchase durable goods. Ideally, many motivations should be incorporated into the model, but in order to provide a rigorous and simple exposition, only the possibility of an income decrease is incorporated into the model. The mathematical derivations are shown in Appendix 2.

Optimal Savings With Perfect Certainty

Zero Real Interest Rate

If a consumer is certain that real income will decrease (growth rate g is negative,) and the consumer faces a real interest rate of zero (not unrealistic for taxable liquid asset holding), the consumer will plan to have equal consumption over the three periods. The amount of savings set aside in period one to allow for the income decreases in periods two and three will amount to:

(1)

At the end of period one, the liquid asset holding accumulated as a proportion of period one income would equal the amount shown in Equation 1. For instance, if a consumer is certain that real income will decrease by 50% between period one and period two, then remain at that level, the optimal amount to save out of period one income is 33.3%. If the time period is years, at the end of year one, liquid holdingss will equal four months income. To express the proportion in the same terms as the usual prescription, it should be converted to a proportion of spending. Year one spending equals two thirds of income, so liquid asset holdings as a proportion of spending equals six months income, which is equal to the

4



typical prescription. The optimal savings as a percent of year one income and consumption is shown in Figure 1, for levels of income decreases ranging from 60% to zero. The real interest rate assumed is zero, so the utility function does not make any difference in the analysis, if the personal discount rate is zero. Only households who were certain that real income would drop 50% between year one and two, then remain at that level, would accumulate savings by the end of year one at the prescribed level to cover six months worth of spending.^b

Figure 1. Optimal Savings as a Percent of Year 1 Income, by Real Growth Rate of Income Between Year 1 and Year 2, Assuming Growth Rate Known with Certainty and Real Interest Rate = 0.



Non-Zero Real Interest Rates

The optimal year one savings as a proportion of year one income can be derived by calculus, and is shown in



Emergency Fund Levels: Is Household Behavior Rational?

Equation 7 of Appendix 2. Given that the real interest rate on liquid assets is usually close to zero, the optimal saving/income ratios obtained will be very close to those obtained from Equation 1 above. The results for other plausible real interest rates on liquid asset holdings, ranging from -1% to 4%, are virtually identical to the results shown in Figure 1 for a range of levels of relative risk aversion. However, an analytical solution for optimal saving is not possible if uncertainty is allowed, especially if different real interest rates for borrowing and saving are assumed. Therefore, a numerial method ("simulations") is used to find the optimal saving/income ratio.

In this section, the impact of the growth rate on optimal savings levels is discussed and illustrated. The value assumed for relative risk aversion is six (Chang, Fan & Hanna, 1992), but results are similar for other plausible values. A graph is produced to help illustrate effects of these parameters by using a numerical simulation technique. In order to focus on scenarios with saving, it was assumed that the consumer faced either constant real income or a negative real income growth rate g with a probability p. The simulations were based on the following assumptions:

- The real interest rate on savings = 1% (e.g., nominal interest rate of 8.4%, subject to 28% tax rate and 5% inflation.)
- The real interest rate on loan = 14.095% (e.g., nominal rate of 19.8% with 5% inflation.)
- Expected utility from all possible borrowing levels (at 14.095%) is compared to expected utility from all possible saving levels (at 1%) and optimal saving/borrowing is that which produces highest expected utility.

The results are similar for other plausible levels of interest rates.

Figure 2. Optimal Savings as a Percent of Year 1 Income, by Probability Income Drops Between Year 1 and Year 1.

Figure 2 shows the result of the simulations based a range of probabilities that real income drops by 50% between year one and two, then remains at the new level during year 3. For a probability of 100% that real income drops by 50%, the results are virtually identical to the analysis

illustrated in Figure 1. As the probability decreases, the optimal amount of savings drops rapidly. If the probability of real income dropping by 50% is 15%, then the household's savings should amount to 25% of annual spending. In a recession, this is possible for some occupational groups, but for many households, the probability of such a drastic decrease in real income is lower than 15%.

Empirical Analysis

The theoretical model of optimal savings described above showed that optimal holdings of emergency fund should be negatively related to expected income growth rate. Few empirical studies have been done to examine the relationship between expected future income growth rate and emergency fund holdings. Using panel data from the 1983 and 1986 Survey of Consumer Finances (SCFs), an empirical test of the theoretical notion on the relationship between expected future income and adequacy of emergency fund holdings was conducted. A total of 2,450 households who were interviewed in both 1983 and 1986 were used in the empirical test with non-probability high income sample excluded.

To determine the effect of expected future income growth rate on adequacy of emergency fund holding, a logistic regression analysis was used. Emergency fund holding was defined as household liquid assets holdings which include values in savings and checking accounts, money market funds, certificates of deposit, stocks, and bonds. Mean levels of household's emergency fund holdings were \$22,499 (in 1986 constant dollar) in 1983 and \$24,589 in 1986 respectively. Households' emergency fund holdings were further defined as adequate (meeting recommended guidelines) if the value of emergency fund reserves exceeded three months of the household's gross income. With this criterion, the data showed that only 37% of the households had adequate emergency fund reserves in 1983 and 37% did in 1986. The dependent variable used in the logistic regression is dichotomous and was set equal to one if the household's emergency fund holding in 1983 exceeded three months of the household's gross income in 1983, and equal to zero otherwise.

Estimation of income expectation variables Since the subjective probability density function of

expectation of future income is not observable, an income prediction equation was estimated using four years of income information (1982-1985) from the 1983 and 1986 SCFs to construct an expected future income rate variable. The predicted future income variable is a theoretical expectation rather than the household's subjective expectation. The theory of rational expectation suggests that households should be able to predict their future income flow based on their demographic characteristics and expectations about future events related to income change. Expected household income is therefore estimated assuming that the household projects its future income according to the current income, current family composition, job status and other socioeconomic factors.

To reduce the effect of year-to-year fluctuations, two new income variables were created based on income from the first two years and the last two years. The income of 1982 and 1983 were used for total income for the household in the first period (1982-1983) and included as an independent variable to predict future income. The incomes from 1984 and 1985 were used to represent total household income in the second period (1984-1985). The expected income of 1984-1985 was estimated by an income prediction equation which uses actual income of 1984-1985 as the dependent variable and the following independent variables measured as of 1983: household size, educational level of the respondent, race, age of the respondent, age squared, occupation, marital status, and job status of the respondent, actual total 1982-1983 household income, total 1982-1983 income squared, and the interaction terms between these variables.

A stepwise regression analysis was used for estimation. With stepwise regression, it is possible to test the potential effects of a large number of variables in an equation, including interaction terms, to obtain the best set of predictors for the dependent variables (Neter, Wasserman, & Kutner, 1989). The common criticisms of stepwise procedures are not relevant, as there was no interest in estimating any particular parameters. In order to obtain the best possible prediction of future income, a number of interaction terms were included in a list of potential regressors, as it was possible that, for instance, the effect of education on income might depend on age. The final step of the regression model consisted of 31 explanatory variables. The R² of the income prediction equation was 0.81, indicating that 81% of the variation in future income can be accounted for by the independent variables. Results of the income prediction regression are can be found in Chang and Hanna (994). The expected future income growth rate was defined as the difference between predicted 1984-1985 income and actual 1982-1983 income devided by 1982-1983 income:

(2)

All four year incomes were converted to 1986 dollars. The expected income growth rates reported were thus the real rate, and did not include the effect of inflation. Table 1 summarizes descriptive statistics of the predicted income growth rate. The mean and median of predicted income growth rate were 17% and 8%, respectively. However, between the 1982-1983 and 1984-1985 period, 28% of the sample could have expected a negative income growth while 10% of the sample could have expected an increase of 50% or more.

Table 1. Descriptive Statistics of Expected Income Growth Rate (n=2,450)

Mean	17%
10 th percentile	-10%
25 th percentile	-1%
median	8%
75 th percentile	25%
90 th percentile	55%

Based on the results from the income prediction equation, six categories indicating different levels of expected income change were created. The mean proportion of sample meeting the three months emergency fund guideline by these six categories was computed. A multiple means comparison test was employed to test if these mean proportions of sample meeting the guideline were significantly different among the six groups. It was expected that households who could have anticipated an income decrease were more likely to hold adequate emergency fund reserves. Furthermore, the mean probability of meeting the recommended emergency holding guideline should increase as the expected income change becomes more negative.

Results of Multiple Means Comparison Test

Results of the multiple means comparisons test are shown The mean probability of sample in Table 2 below. meeting the three months emergency fund guideline declined from 54% for those expecting a 50% or more decreases in real income, 50% for those expecting a decrease in real income between 10% to 50%, 47% for those expecting a decrease in real income between 5% to 10%, 44% for those expecting a decrease in real income less than 5%, to 34% for those who did not expect future real income to decrease. The mean probability of meeting the guideline for households who did not expect a decrease in future income was significantly lower than households expecting some decreases in their future income. Although other factors may affect emergency fund holding, the bivariate empirical relationship between the likelihood of having adequate emergency fund reserves and expected income drop rate somewhat confirms the theoretical model^c.

Table 2.

Multiple Means Comparisons Test for Probability of Meetinng 3 Months Emergency Fund Guideline by Different Income Growth Rates

Groups	Mean probability of meeting the guideline	% of households in growth category
(1) Income growth rate < -50%	54%	0.5%
(2) $-50\% \le$ Income growth rate $< -25\%$	50%	2.9%
(3) -25% ≤ Income growth rate < -10%	50%	6.8%
(4) $-10\% \le$ Income growth rate $< -5\%$	47%	6.3%

$(5) -5\% \le$ Income growth rate < 0	44%	11.5%
(6) Income growth rate >=0	34%	72.0%
Note:		

C

. Group (6) is significantly different from groups (2), (3), (4), and (5) at .05 level

. Group (5) is significantly different from groups (2), (3), and (4) at .05 level

Conclusions

Previous empirical research on emergency fund holding of households have made implicit or explicit assumptions that the typical prescription of having liquid assets equal to three to six months worth of spending was valid for most households. One might then conclude that most U.S. households were mistakenly not holding adequate levels of liquid assets. The empirical analysis presented in this paper shows that 63% of U.S. households did not have enough liquid assets to cover three months of income. However, the original theoretical analysis presented in this paper suggests that only those who have a 15% chance that household income will drop by at least 50%, or some similar combination of probablity and magnitude of drop, should hold that level of liquid assets. The empirical patterns of households meeting the three month standard suggest that households expecting a decrease in their real future income were significantly more likely to hold adequate emergency fund reserves. The probability of meeting the three month standard increases as the expected income change becomes more negative. Given that only 28% of the household rationally could have expected a decrease in real income, the majority of the households (63%) may rationally have chosen not to have the recommended emergency fund reserve. This paper ignores other motives for holding liquid assets, so the results should be interpreted cautiously.

Consumer education related to holding emergency funds should focus on specific motivations for holding liquid assets. Garman and Forgue (1997, 77-78) provide a good approach to this issue, but the analysis should be taken further. The fact that 63% of households do not follow a common prescription might suggest vigorous efforts at education, but further research to refine that prescription

and tailor it to the situation of a specific household would be useful. In the future, perhaps computer expert systems could help individual consumers decide on optimal levels of emergency funds.

Endnotes

- a. The theoretical exposition presented is the same as presented in Hanna, Chang, Fan, and Bae (1993). The theoretical derevation was the work of Fan, Chang and Hanna.
- b. Chang and Lindamood (1993) showed that less than 10% of U.S. households had a chance of an income drop of 50% or more.
- c. A multivariate logit of whether the 3 month guideline was met, with dummy variables for the income growth categories as well as demographic variables, did not show significant effects for the income growth variables. However, it is likely that the demographic variables are so related to expected income growth that they take away the effects of expected income growth.

Appendix 1

Results of Empirical Studies on Household Emergency Fund Holding

Study	Emergenc y fund definition	Emergency fund guidelines and % of households <i>NOT</i> meeting the guideline
Chang & Huston (1995) 1983 & 1986 SCF ⁵	Intermed- iate emergency fund [«]	3 months gross household income → 68% in 1983 and 68% in 1986
Chang (1995) 1983 & 1986 SCF	Compre- hensive emergency fund ^β	3 months gross household income → 63% in 1983 and 63% in 1986
DeVaney (1995) 1977 & 1989 SCF	Compre- hensive emergency fund ^β	3 months gross household income → 66% in 1977 and 65% in 1989
Hanna & Wang (1995) 1990-91 CES ⁿ	Compre- hensive emergency fund ^β	3 months spending $\rightarrow 69\%$

Hanna, Chang, Fan & Bae (1993) 1990-91 CES	Liquid assets [¥]	3 months pretax income \rightarrow 74% 3 months take home income \rightarrow 71% 3 months spending \rightarrow 73% 6 months pretax income \rightarrow 82% 6 months take home income \rightarrow 79% 6 months spending \rightarrow 81%
Prather (1990) 1983 SCF	Liquid assets ⁸	6 months spending \rightarrow 71%
Johnson & Widdows (1985) 1977 & 1983 SCF	Quick emergency fund ^e Intermed- iate emergency fund ^e Compre- hensive emergency fund ^β	2 months gross household income → 58% in 1977 and 73% in 1983 6 months gross household income → 79% in 1977 and 89% in 1983 2 months gross household income → 56% in 1977 and 67% in 1983 6 months gross household income → 77% in 1977 and 84% in 1983 2 months gross household
		2 income → 51% in 1977 and 64% in 1983 6 months gross household income → 71% in 1977 and 81% in 1983

Notes:

- α. Intermediate emergency fund = amount in checking, savings accounts, money market funds and accounts, CDS and savings certificates.
- β. comprehensive emergency fund = intermediate emergency + amount of stocks and bonds.
- γ. liquid assets = amount in checking accounts, brokerage accounts, savings account of banks, savings & loans, credit unions, amount in stocks, bonds, mutual funds, and amount in US savings bonds.
- liquid assets = amount in checking accounts, money market funds, savings and call accounts, amount of 40% of all stocks and bonds, and amount of 30% of mutual funds.
- e. quick emergency fund = amount in checking, savings accounts, and money market funds.
- ζ . SCF = U.S. Surveys of Consumer Finance, sponsored by the Federal Reserve Board
- η CES = U.S. Consumer Expenditure, sponsored by the Bureau of Labor Statistics of the U.S. Department of Labor.

Emergency Fund Levels: Is Household Behavior Rational?

Appendix 2

Mathematically, the problem can be formulated as shown in Equation 1.

(1)

The constraints are:

$C_1 = I - S_1$	(2)
$C_2 = (1+g)*I + (1+r)*S_1 - S_2$	(3)
$C_{2a} = I + (1+r)*S_1 - S_2$	(4)
$C_3 = (1+g)*I + (1+r)*S_2$	(5)
$C_{3_{2}} = I + (1+r)*S_{2}$	(6)

Variables:

T = Total three period utility

I = Y ear 1 income

 $I_2=(1\!+\!g)*I$ (if income increases in that year), otherwise, Year 2 income = Year 1 income

 $C_1 = Consumption in year 1$

 S_1 = The amount of savings in year 1

 C_2 = Consumption in year 2 if real income in year 2 increases

 C_{2a} = Consumption in year 2 if real income in year 2 does not increase S_2 = The amount of savings in year 2

 $C_3 = Consumption in year 3 if real income in year 2 increases$

 C_{3a} = Consumption in year 3 if real income in year 2 does not increase g = Growth rate in real income (negative number means decrease rate in real income)

r = Real interest rate (Note that r may be higher for S<0, i.e., borrowing, than for S>0)

P = Probability that real income decreases

 ρ = personal discount factor. (This might vary.)

A consumer may discount utility from future consumption because of the possibility that he/she may not be alive then, or because of other possible changes in capacity to derive utility from consumption. Young adults have very low risks of death, so this source of discounting should not be important for them. For analysis of savings/credit, the approximate effect of a nonzero personal discount rate is to reduce the real interest rate in the optimal solutions shown below, so that instead of an interest rate of r, the consumer in effect faces an interest rate of r- ρ . For the remainder of this paper, ρ is assumed to equal zero. If ρ is positive rather than zero, a consumer would save less or borrow more for any given set of values of other parameters.

Most studies of intertemporal consumption have used a constant elasticity utility function (Hurd 1989) which is time separable additively. See Fan, Chang and Hanna(1993) or Hanna, Fan and Chang (1995) for a simple exposition of utility functions for intertermporal choice and arguments as to why a plausible level of relative risk aversion is 6.0.

By combining intertemporal consumption analysis with risk aversion, we can obtain the optimal amount of saving in terms of year 1 income, interest rate, income growth rate, and probability of that income increases. If there is certainty, equation 7 shows the optimal amount of savings in period 1 for the three period model in which income increases by a growth rate g between period 1 and 2, then remains at that level in period 3.

(7)

References

- Asinof, L. (October 9, 1992). Paying down your debt can boost your returns. *The Wall Street Journal*. V. LXXIII(254), C1.
- Bae, M.K. (1992). Analysis of Household Spending Patterns. Ph.D. dissertation, The Ohio State University.
- Chang, Y. R. & Hanna, S. (1994). Determinants of household expected real income growth in the U.S.A. *Journal of Consumer Studies and Home Economics*, 18, 315-329.
- Chang, Y. R., Fan, J. X. & Hanna, S. (1992). Relative risk aversion and optimal credit use with uncertain income. In Virginia A. Haldeman (Ed.), *The Proceedings of the 38th* Annual Conference of American Council on Consumer Interests, 14-22.
- Chang, Y. R. & Hanna, S. (1994). Determinants of household expected real income growth in the U.S.A., *Journal of Consumer Studies and Homes Economics*, 18, 315-329.
- Chang, Y. R. & Lindamood, S.(1993). Factors related to the risk of household income variability, *Financial Counseling* and Planning, 4, 47-66.
- Dreze, J. & Modigliani, F. (1972). Consumption decision under uncertainty. *Journal of Economic Theory*, 5(3), 308-335.
- Fan, X.J., Chang, Y.R. & Hanna, S. (1993). Real income growth and optimal credit, *Financial Services Review*, 3(1), 45-58.
- Garman, E. & Forgue, R. (1997). Personal Finance. Boston, Mass: Houghton Mifflin Company.
- Greninger, S. A., Hampton, V. L., Kitt, K. K. & Achacoso, J. A. (1996). Ratios and benchmarks for measuring the financial well-being of families and individuals. *Financial Services Review*, 5(1), 57-70.
- Griffith, R. (1985). Personal financial statement analysis: a modest beginning. In G. Langrehr (Ed.), *The Proceedings* of the Third Annual Conference of the Association for Financial Counseling and Planning Education, 123-131.
- Grossman, S. & Shiller, R. (1981). The determinants of the variability of stock market prices. *American Economic Review: Papers and Proceedings*, <u>71</u>(2), 222-227.
- Hall, R. (1988). Intertemporal substitution in consumption. *Journal of Political Economy*, 96(2), 339-357.
- Hanna, S. (1989). Risk aversion and optimal insurance deductibles. In Mary Carsky (Ed.), *The Proceedings of the* 35th Annual Conference of American Council on Consumer Interests, 141-147.
- Hanna, S., Fan, X.J. & Y. R. Chang (1995). Optimal life cycle

savings, Financial Counseling and Planning, 6, 1-15.

- Hanna, S., Chang, Y.R., Fan, X.J. & Bae, M. (1993). Emergency fund levels of households: is household behavior rational?, *Proceedings of The American Council* on Consumer Interests 38th Annual Conference. 215-222.
- Hanna, S. & Wang, H. (1995). The adequacy of emergency

funds to cover household expenditures *Consumer Interests* Annual, 41, 224-225.

- Hey, J. (1979). *Uncertainty in Microeconomics*. New York: New York University Press.
- Hubbard, R. G., Skinner, J. & Zeldes, S. P. (1995). Precautionary saving and social insurance. Financial Research Center Working Paper 3-95, Wharton School, University of Pennsylvania.
- Hurd, M. (1989). Mortality risk and bequests. *Econometrica*, *57*(4), 779-813.
- Johnson, D. & Widdows, R. (1985). Emergency fund levels of households. In K. Schnittgrund (Ed.), *The Proceedings of* the American Council on Consumer Interests 31th Annual Conference, 235-241.
- Kinsey, J. & Lane, S. (1978). The effect of debt on perceived household welfare. *The Journal of Consumer Affairs*, 12(1), 48-62.
- Leland, H. (1968). Saving and uncertainty: The precautionary demand for saving. *Quarterly Journal of Economics*, *LXXXII*(3), 465-473.
- Levhari, D. & Srinivasan, T. (1969). Optimal savings under uncertainty. *Review of Economic Studies*, <u>36</u>(106), 153-163.
- Lindqvist, A. (1981). A note on the determinants of household saving behavior. *Journal of Economic Psychology*, *1*, 39-57.
- Mirman, L. (1971). Uncertainty and optimal consumption decisions. *Econometrica*, 39(1), 179-185.
- Neter, J., Wasserman, W., & Kutner, M. H. (1989). *Applied Linear Regression Models*, Richard D. Irwin, Homewood, IL.
- Prather, C. (1990). The ratio analysis technique applied to personal financial statements: Development of household norms. *Financial Counseling and Planning*, 1, 53-69.
- Sandmo, A. (1970). The effect of uncertainty on saving decisions. *Review of Economic Studies*, <u>37</u>(111), 353-360.
- Salyer, K. (1988). The characterization of savings under uncertainty: The case of serially correlated returns. *Economic Letters*, 26(1), 21-27.
- Sibley, D. (1975). Permanent and transitory income effects in a model of optimal consumption with wage income uncertainty. *Journal of Economic Theory*, <u>11</u>(1), 68-82.
- U.S. Bureau of Labor Statistics [USBLS] (1986). Consumer Expenditure Survey: Interview Survey, 1982-83. Bulletin 2246.

cite as:

Chang, Y. R., Hanna, S. & Fan, J. X. (1997). Emergency fund levels: Is household behavior rational? *Financial Counseling and Planning*, 8(1), 47-55.