

# The Impact of Salary Growth, Inflation, Employee Age, and Career Length on the Relative Desirability of Pension Fund Type

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*The purpose of this paper is to examine how the variables of salary growth rate, inflation, employee age, and career length affect the relative desirability of defined contribution (DC) plans compared to defined benefit (DB) plans. High salary growth rate and employee age favor DB pensions. A longer career favors DC pensions. The impact of inflation is theoretically ambiguous although an example indicates that high rates of inflation favor DC plans. This study is of interest to people with multiple job offers that include both DC and DB pensions.*

**KEY WORDS:** *pensions, personal finance, retirement planning*

Pension plans typically come in two distinct forms: defined benefit (DB) and defined contribution (DC). If a person is considering a job offer from a firm, the terms of the pension plan as well as the salary and all other benefits and perquisites should be a critical part of the decision of whether or not to accept the job offer. If a person is considering more than one job offer, some offers may include DC pensions and others may include DB pensions. In this situation, not only are the terms of the pension plans critical, but the sensitivity of the benefits to various parameters becomes critical.<sup>a</sup>

An employee evaluating a pension plan would most likely first inquire as to the likely pension benefit. The second question should deal with the risk of that pension benefit where risk is defined as the variability of the pension benefit (McLeod, Moody & Phillips, 1992/93; Woerheide & Fortner, 1994). The purpose of this paper is to focus on the relative impact on the final pension benefit from changes in one's salary growth rate, the inflation rate, an employee's age, and an employee's projected time until retirement.<sup>b</sup>

## Why Comparisons of DC and DB Pensions Are Difficult

Anyone familiar with the literature discussing DB and DC plans knows the debate as to which plan is "better" is irresolvable for two reasons. The first is that pension

plans may incorporate much more than just a prospective retirement benefit. They may also include such items as survivor protection, disability protection, retiree medical benefits, early retirement incentive programs, inflation adjustments, social security offset, death benefits, etc. (Banker's Trust Company, 1980; Woerheide & Fortner, 1991; Kahl & Williams, 1994). Although most of these benefits could be offered independently of a pension plan, some are an integral part of a pension. In addition, pensions likely will differ with respect to vesting requirements. The second reason is that there are different types of risk in the two types of pensions.

As both these benefit differences and risk differences are difficult to quantify, the present value of the pension benefit component of a compensation package can never be accurately valued.<sup>c</sup> Thus, a definitive solution to the question of which type of pension plan is better is unlikely to be developed.

Having now established that comparisons of the two types of pensions is a nearly moot topic, I would like to proceed to compare them! Mathematical models for the pension benefit under each type of pension plan will be defined first. The ratio of the pension benefit under a DC plan relative to the pension benefit under a DB plan is then formulated. How this ratio of relative benefits is

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affected by changes in several of the parameters that determine the pension benefit is then examined. This analysis will not resolve the dispute as to which type of pension is better, but it will provide some information that financial planners should be familiar with, and prospective employees should understand if they have the opportunity to choose between or among pension plans.

**Models for a DB and a DC Plan**

In a typical DB plan the pension benefit is a function of "final" salary, years of employment, and a fixed percentage awarded per year of employment.<sup>d</sup> Mathematically, this could be expressed as:

$$PB(DB) = n * c * S * (1 + g)^{n-1} \tag{1}$$

where

- PB(DB) = annual pension benefit under a DB pension
- n = number of years of expected employment (i.e., career length)
- c = fixed percentage awarded per year employed
- S = current or initial annual salary
- g = the mean annual growth rate in salary

As an example of equation (1), suppose a DB pension promises 2% of a person's final salary for each year of employment (c = .02), the person works for 30 years (n = 30), his current salary is \$30,000 (S = \$30,000), and he expects his mean annual salary growth rate to be 5% (g = .05). This person's annual pension benefit will equal 60% (n\*c) of his final salary of \$123,483.80 (S\*(1+g)<sup>n-1</sup>), or \$74,090.28.

The DC plan is more complex. The annual benefit under a DC pension plan is a function not only of the employee's salary growth rate, but also of the investment yield on the pension fund assets, and the price of a lifetime annuity contract upon retirement. The pension benefit can be expressed as

$$PB(DC) = \left( \sum_{t=1}^n k * S * (1 + g)^{t-1} * (1 + i)^{n-t} \right) / AF \tag{2}$$

where

- k = percentage of salary contributed to the pension account<sup>e</sup>
- i = the investment yield
- AF = the annuity factor used to convert the pension account into a lifetime annuity upon retirement

The annuity factor is an actuarial based computation that is a function of mortality rates, investment yield, and a loading factor (i.e., the fee charged to cover the administration of an annuity). The annuity factor can be defined as

$$AF = (1 + f) * (1 / L_{rt}) * \sum_{q = rt+1}^{100} L_q / (1 + i)^{q-rt} \tag{3}$$

where

- L<sub>q</sub> = number of people in a radix group alive at age q
- L<sub>rt</sub> = number of people in a radix group alive when the employee retires and the annuity starts
- f = loading factor charged for an annuity
- rt = age at which a person retires

As an example of a DC pension, suppose 12% of an employee's salary is contributed to a pension plan (k = .12), the investment yield on the pension fund is 7% (i = .07), the annuity factor is based on the Commissioner's 1980 Standard Ordinary Table, the employee retires at age 65 (rt = 65), and the loading factor is 5% (f = .05). We will assume the same starting salary, salary growth rate, and career longevity as in the previous example. In this case, the pension will be \$72,510.46 (58.72% of the final salary of \$123,483.80).

For analytical purposes, we can judge the impact of variables on the relative desirability of the two types of pension funds by focusing on how each variable affects the ratio of the pension benefit under a DC plan to the benefit under a DB plan. To keep the equations simple, the term AF will be used to represent the annuity factor in all of the computations where the specification of that function is immaterial. The resulting ratio can be simplified to<sup>f</sup>

$$R = (K / AF) * \sum_{t=1}^n (1 + g)^{t-1} * (1 + i)^{n-t} \tag{4}$$

where

- R = PB(DC)/PB(DB) = the value of the expected pension benefit under a DC plan relative to that available under a DB plan
- K = k/(n \* c)

If the two pension plans being compared are such that the parameters produce a value of R greater than 1, then the DC plan would be preferable with respect to expected benefits. If the value of R were less than 1, then the DB plan would be preferable. The actual

choice, of course, would require knowledge not only of the expected benefits, but also of the riskiness of those benefits.<sup>g</sup> Our interest in this paper is in *relative changes* in R and *not in its absolute or risk adjusted value*.

**The Impact of Changes in the Salary Growth Rate**

An examination of equations (1) and (2) make it abundantly clear that under either type of pension the larger the growth rate in one's salary, the larger one's pension benefit. The real question is which pension is more sensitive to increases in the salary growth rate. The partial derivative of the ratio of benefits with respect to the salary growth rate, is negative because

$$\frac{\partial R}{\partial g} = \frac{(K/AF)^n}{t=1} (t-n)(1+g)^{t-n-1} * (1+i)^{n-t} \quad (5)$$

the term (t-n) in the summation is less than or equal to zero for each value of t and all other components are positive. The negative derivative means that the higher one's expected growth rate in salary, the larger the value of a DB plan benefit becomes relative to the benefit under a DC plan, other things equal.

Some readers have found this result counter-intuitive (and others find it trivial). After all, the faster the growth in salary, the larger the dollar value of contributions to a DC type plan! Although this is true, one must remember that the larger the salary growth rate, the smaller the pension accumulation from the early contributions relative to the final salary number. To continue the previous example of the benefit under a DC plan, if all the numbers are the same except that the salary growth rate is changed from 5 to 6%, then the pension benefit climbs to \$88,611. However, the final salary will be \$162,551.40, and the pension benefit will be 54.5% of the final salary. The pension benefit under the DB plan remains constant at 60% regardless of the salary growth rate.

In the above equations, a constant salary growth rate was assumed. If we relax that assumption and allow annual salary increments to vary without altering the geometric mean growth rate, then an additional element of risk in a DC pension becomes apparent. Variations in annual salary growth rates do not affect the benefit under a DB plan. So any combination of salary increments which produce the same geometric mean annual growth rate over the life of a job will produce the same pension benefit. But variations in salary

increments will affect the pension benefit under a DC plan. Lower than average salary increases in the early years will produce a lower DC benefit than that provided by a constant salary growth rate, and higher than average salary increases in the early years will produce a higher DC benefit.<sup>h</sup>

**The Impact of Changes in the Inflation Rate**

The impact of unanticipated changes in the inflation rate is much more difficult to analyze because the rate of inflation affects both investment yields and salary growth rates. For analytical purposes, let us assume that a person's salary growth rate is itself the sum of two numbers. The first is compensation for the net increase in the value of human capital (i.e., merit) and the second is compensation for inflation, i.e.,  $g = h + p'$  where h equals annual compensation for net increases in the value of human capital, and p' equals annual compensation for inflation. Similarly, the investment yield can be expressed as the sum of a risk adjusted rate of return plus an inflation premium, i.e.,  $i = r + p''$  where r equals the risk-adjusted, inflation free rate of return, and p'' equals incremental expected yield due to inflation.

If h and r are treated as fixed, then three possible cases should be considered with respect to the relationship between p' and p''. The first is that inflation increases salary growth rates by a larger amount than investment yields (i.e.,  $p' > p''$ ).<sup>i</sup> We have already established that higher salary growth rates favor DB plans, and it is simple to show that lower investment yields also favor DB plans. Thus, this case increases the relative attractiveness of DB plans. The second case is that inflation increases investment yields by a larger amount than salary growth rates (i.e.,  $p' < p''$ ). As this is simply the flip side of the first case, it is not surprising that the mathematical solution results in making DC plans relatively more attractive.

The third case is that the incremental impact of inflation on salary increases and on investment yields is the same (i.e.,  $p' = p'' = p$ ). If so, the ratio of pension benefits can be restated as

$$R = \frac{(K/AF(p))^n}{t=1} (1+h+p)^{t-n} * (1+r+p)^{n-t} \quad (6)$$

where  
 AF(p) = annuity factor with the inflation rate explicitly incorporated

The partial derivative of the relative value of benefits with respect to the inflation rate is then defined as

$$\frac{\partial R}{\partial \phi} = M_1 * \frac{\partial M_2}{\partial \phi} + M_2 * \frac{\partial M_1}{\partial \phi} \quad (7)$$

where

$$M_1 = K/AF(p)$$

$$M_2 = \sum_{t=1}^n (1+h+p)^{t-n} * (1+r+p)^{n-t}$$

The values for  $M_1$ ,  $M_2$ , and the partial derivative of  $M_1$  are always positive. The derivative of  $M_2$  can be shown to equal

$$\frac{\partial M_2}{\partial \phi} = \sum_{t=1}^n (1+h+p)^{t-n-1} * (1+r+p)^{n-t-1} * [(1+h+p) - (1+r+p)] \quad (7a)$$

A sufficient condition for the above derivative to be positive is that the expression in brackets be positive. This would mean  $(1+h+p)$  is greater than  $(1+r+p)$ , or more simply that  $h > r$ .

As a practical matter, historical investment yields on pension funds have consistently exceeded historical averages for salary increases. Furthermore, most pension fund actuaries assume investment yields in excess of projected salary increases with a model spread of 2% (Wyatt, 1981). Unfortunately, if we do assume that  $r + p$  is greater than  $h + p$ , then all the terms in equation (7a) are negative and the sign of the derivative in equation (7) is ambiguous.

However, we can examine the value of equation (7) with a set of plausible values for the different parameters. If we assume the parameter values for the two pension benefit examples in section II, then we obtain the following values for the terms in equation (7):  $M_1 = .0245$ ,  $M_2 = 39.9685$ ,  $\frac{\partial M_1}{\partial \phi} = .5989$ , and  $\frac{\partial M_2}{\partial \phi} = -11.3171$ . Plugging these values into equation (7), the derivative equals 23.6598. Hence, for this example the derivative is positive and therefore when inflation affects salary growth rates and investment yields symmetrically, higher rates of inflation make DC type plans relatively more attractive. It should be noted that this example is not an unique estimation but is consistent with calculations based on a multitude of combinations of  $N$ ,  $r + p$ , and  $h + p$  (Woerheide & Fortner, 1994).

So, which of the above three cases is more likely? Although unanticipated changes in the inflation rate inversely affect market values of pension fund assets, it would also appear that unanticipated changes in inflation become anticipated in future periods.<sup>1</sup> Thus, over one's working career initial losses (gains) in portfolio values due to inflation would be offset by higher (lower) rates of return in future time periods. A similar argument can be made in the case of wages, at least in the presence of multi-year wage contracts. As an equilibrium condition, the most likely case may be that the impact of inflation is symmetrical for the two parameters (i.e.,  $p' = p''$ ). Thus, it would appear that higher inflation rates would favor DC plans.

### The Impact of Employee Age

The impact of an employee's current age on the relative value ratio can be computed by assuming that the retirement date is fixed and taking the difference between the relative value ratios when an employee has  $n + 1$  years left to work and when he has  $n$  years to work. The assumption that the retirement date is fixed (e.g., retirement at 65) means that the annuity factor would be the same for both relative value ratios. The difference in relative value ratios can then be calculated and simplified to<sup>k</sup>

$$(R_{n+1} - R_n) / r = (K/AF) * \left[ 1 + \sum_{t=1}^n [(1+g)^{t-n-1} * (1+i)^{n+1-t} - (1+g)^{t-n} * (1+i)^{n-t}] \right] \quad (8)$$

A sufficient condition for this difference to be positive is that  $(1 + i)$  be greater than  $(1 + g)$ . In other words, if the expected investment yield is greater than the expected annual growth rate in salary (which has previously been suggested to be the case), then the earlier one starts contributing to a pension (i.e., the younger the employee when starting a new job) the more likely a DC plan would be preferable.

### The Impact of Career Length

In this section we focus on the case in which a worker's current age is fixed and the retirement age is allowed to vary. On the surface, this might sound like the same issue just discussed in the prior section. However, there are some differences. In this section, we focus on the fact that an employee may have some uncertainty about the age at which he or she might retire. In the previous

section, we assumed that the retirement age was known with certainty, and ascertained that younger starting workers are more likely to prefer DC plans, and older starting workers are more likely to prefer DB plans.

Analyzing the impact of career length is more complex than analyzing the impact of a worker's age on the relative value of the two types of pensions because the annuity factor is no longer constant. As before, the analysis can be done by taking the difference in the relative value ratios if a worker plans to work  $n + 1$  years versus  $n$  years. This difference can be expressed as

$$(R_{n+1} - R_n | \text{current age is fixed}) = K \left[ \prod_{t=1}^n (1/AF_{t+1}) * (1+g)^{t(n+1)} * (1+i)^{(n+1)-t} - \prod_{t=1}^n (1/AF_t) * (1+g)^{t*n} * (1+i)^{n-t} \right] \quad (9)$$

This expression can be simplified to show that a sufficient condition for it to be positive is that  $(AF_n/AF_{n+1})$  be greater than  $(1+g)/(1+i)$ . It is always the case that the ratio of annuity factors will be greater than one.<sup>12</sup> It now has been suggested twice that as an empirical matter the investment yield will be greater than the salary growth rate (i.e.,  $i > g$ ). Therefore, the ratio of  $(1 + g)$  to  $(1 + i)$  will be less than one. Thus, equation (9) will always be positive and therefore when a worker contemplates a longer working career, the relative value of the DC pension plan increases. One implication of this is that the type of pension plan a worker has could ultimately influence the decision of when this person retires (Woerheide, 1992).

### Summary and Conclusions

The purpose of this paper has been to identify the conditions which cause a DC type pension to become more favorable relative to a DB type pension. The variable analyzed was the ratio of the projected annual pension benefit under a typical DC plan to the projected annual pension benefit under a typical DB plan. The purpose was not to determine which type of pension is better in absolute terms because the answer to that question depends both on the specific parameters for each type of pension and on difficult to quantify pension fund characteristics.

Three conclusions are clear from this analysis. The first is that the two parameters for which higher values favor a DB type pension are salary growth rate and employee age. That is, the higher the projected salary growth rate or the older an employee starting a new pension, ceteris paribus, the more favorable a DB type pension becomes. The second is that the longer an employee plans to work, ceteris paribus, the more favorable the DC type pension becomes. The third is that the impact of the inflation rate is theoretically ambiguous. An example provided in this paper and empirical evidence in the literature indicates that for plausible values of the parameters involved, higher rates of inflation favor DC types of pensions.

### Endnotes

- a. *The primary audience for this paper is, as indicated, people comparing job offers which include DC and DB pensions. Another audience would be employees whose companies are considering the introduction of a pension plan where one has not existed.*
- b. *For any comparison of pension benefits to be valid, ceteris paribus must apply. In this case, other things equal include starting salaries, expected salary growth rates, employee contribution rates, etc.*
- c. *The difficulty in either understanding or comparing pensions is not just an academic question. A study by the GAO found that "millions of workers do not understand their plans' early and normal retirement eligibility requirements" (United States General Accounting Office, 1987, p. 1). Topolnicki (June, 1989, p. 187) noted that more than two million workers have received lower than expected benefits as a result of "reversions" by DB pensions. So there is a strong argument that many workers really do not know the value of their pension plans.*
- d. *Obviously, a substantial number of variations exist in the structure of pension plans. It is not the purpose of this paper to explore the multitude of variations among plans. For simplicity, it is assumed in the model presented that "final" salary means the salary during the last year of employment. "Final" salary in many plans is the average salary during the last few years of employment (typically the last three or five years), and in some cases "final" salary is actually the average salary for all years employed. The interpretation of "final" is immaterial to the results presented herein.*
- e. *As we assume equal starting salaries in this analysis, the starting salary of the job with the DC pension is stated as net of any employee contribution to the pension fund.*
- f. *To simplify the presentation of this paper, many of the derivations are omitted from the text, and some of the formulas discussed are also omitted. All derivations and formulas are available from the author upon request.*
- g. *The multitude of other components associated with pension plans would also be relevant. But to simplify this analysis, they are being ignored.*

- h. *At the extreme, the worst scenario would be to have all the salary increases except the last equal to zero. The best possible case would be to have all the salary increases except the first be equal to zero. (Both these examples assume the same geometric mean annual growth rate.)*
- i. *Keep in mind that changes in the inflation rate in this discussion refer to unanticipated inflation.*
- j. *Pension funds invest primarily in stocks and bonds. Increases in the interest rate resulting from incorporation of currently observed unanticipated inflation, clearly reduce the current value of bonds but increase reinvestment opportunities. The impact of unanticipated inflation on stocks is not nearly as clear, as a rather lengthy literature on this subject illustrates.*

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