Optimal Life Cycle Savings

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How much should a family save for retirement? A prescriptive life cycle savings model is presented. Scenarios are developed with simulations to provide implications for personal financial planning. The percent of income to save today depends on the expected lifetime non-investment income pattern. Households who are sure that their real incomes will increase substantially in the future may be rational in not starting to save for retirement until 25 years before retirement. With uncertain future incomes and retirement ages, saving early may be rational. A computer program based on this model has been used in financial planning classes.
KEY WORDS: *retirement planning, investment, saving, life cycle model, risk tolerance*

Financial educators and planners try to help consumers determine how much to save for retirement. Advice to consumers is generally put in terms of whether a savings plan will achieve an arbitrary level of living for the consumer during retirement. Common advice is that consumers should start saving 10% of their income as soon as they start working (e.g., Gottschalk, 1994). Given the power of compound interest, some financial writers stress the idea that a large amount of saving early in one's adult life is more valuable than waiting to start saving until middle age (Willis, 1988). However, advice given by financial planners and educators has no rigorous basis in economic theory. What insights for financial planning can be provided by a rigorous prescriptive model of life cycle savings?

The procedure for achieving feasible savings and consumption plans is relatively straightforward. For instance, Duncan, Mitchel and Morgan (1984) analyzed the savings ratio needed to achieve a level of living in terms of standard types of retirement level of living goals, that is, a particular replacement rate. Duncan, et al. (1984) used the assumption that one should seek to maintain during retirement the pre-retirement consumption level. The life cycle model contains the assumption that a consumer seeks to maximize utility from consumption over a lifetime (Ando & Modigliani, 1963). If a consumer faces a real interest rate of zero and does not discount future consumption, he or she would seek to have equal consumption over the life span. If these extreme simplifying assumptions are relaxed, implications of the life cycle model are much more complex.

Although the life cycle savings model does not seem to describe consumer behavior well at the household level (e.g., Thaler, 1990) or at the aggregate level (e.g., Mokhtari, 1990), it is the only rigorous model designed to provide a prescriptive answer to the question of how much a consumer should save. In this article, a simplified version of a *prescriptive* life cycle model is first presented, then a more realistic and complex version is developed. Several scenarios are developed and implications for personal financial planning are presented. There are many simplifying assumptions made, such as not considering purchase of housing and other durable items. However, the results presented should give educators and planners insight into what to

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prescribe for any particular family in terms of saving and credit behavior.

The presentation below contains six scenarios to illustrate the prescriptive life cycle model. The first two are based on an extremely simplistic model, in which all initial assets and real non-investment income during the consumer's lifetime are added together to compute total wealth. Based on the concept of decreasing marginal utility of consumption, if the real interest rate faced by the consumer is zero, and the consumer is certain to live to a particular age, the optimal spending (consumption) each year will be the total wealth divided by the number of years left to live. With an uncertain lifetime and a nonzero real interest rate, finding the optimal consumption is more complex. The basic theoretical model is presented in Appendix A, and related theoretical issues are discussed in Appendix B. Scenarios 4-6 illustrate some implications of the prescriptive life cycle model. In all of these scenarios, optimal consumption grows at a rate that is related to the real interest rate faced by the consumer.

For all six scenarios, the amount to save each year is based on the difference between income and the optimal consumption. If income is greater than optimal consumption, the consumer saves that year. If income is less than optimal consumption, the consumer dissaves. If net worth is zero or negative, dissaving implies borrowing. If net worth is positive, dissaving implies taking funds out of assets. Net worth typically increases until retirement, then decreases. However, for scenarios with rapidly increasing real income, net worth may initially decrease as the consumer dissaves.

A Simple Life Cycle Model

Scenario 1: Constant Real Income Until Retirement

In Figure 1, the household's real income is assumed to be $20,000 per year from age 25 until age 64. The earner retires when he/she turns 65. At that point, the pension will be $10,000 per year until death at age 100. The total lifetime income (human capital) is $1,160,000. The average lifetime income is $1,160,000/76 years = $15,263 per year. According to the simple version of the life cycle model, annual spending should be equal to average lifetime income.

Each year before retirement, the household should spend $15,263. Each year, $4,737 ($20,000 - $15,263) is added to a retirement savings account. The annual amount saved is 23.7% of annual income (Figure 2.) After retirement, the household will need to withdraw $5,263 each from retirement savings, which, when added to the pension of $10,000 per year, will allow spending to remain constant at $15,263 per year.

The household is assumed to have zero retirement savings at the beginning. At the end of the first year, the balance of retirement savings will be $4,737. Each year, the balance will increase by that amount in real terms. As Figure 3 shows, retirement savings will increase steadily until age 65, then decrease steadily.

Figure 1.
Scenario 1: Optimal Consumption Plan from Age 25 to 100, Assuming Constant Real Income of $20,000 Per Year, Then Pension of $10,000 Per Year Until Age 100, Assuming Real Interest Rate of 0% and No Personal Discounting.

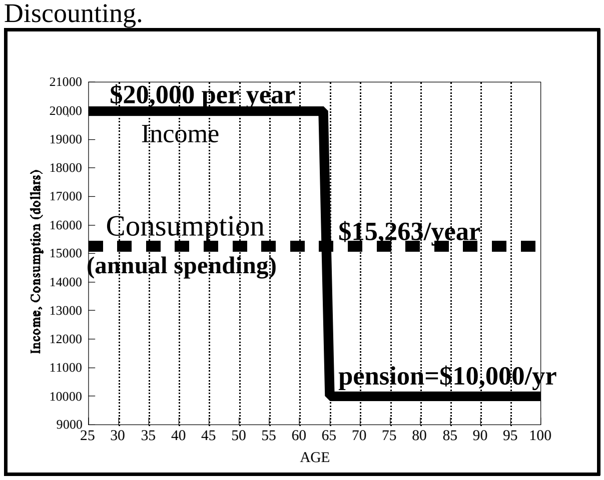


Figure 2.
Scenario 1: Optimal Saving Plan from Age 25 to 100, Assuming Constant Real Income of $20,000 Per Year Until Retirement, Then Pension of $10,000 Per Year Until Age 100, Assuming Real Interest Rate of 0% and No Personal Discounting.

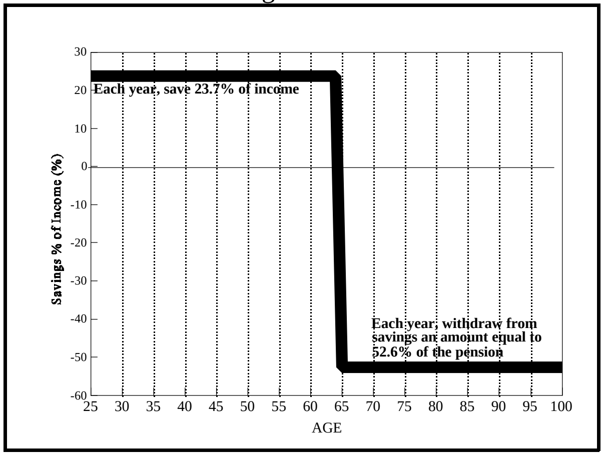
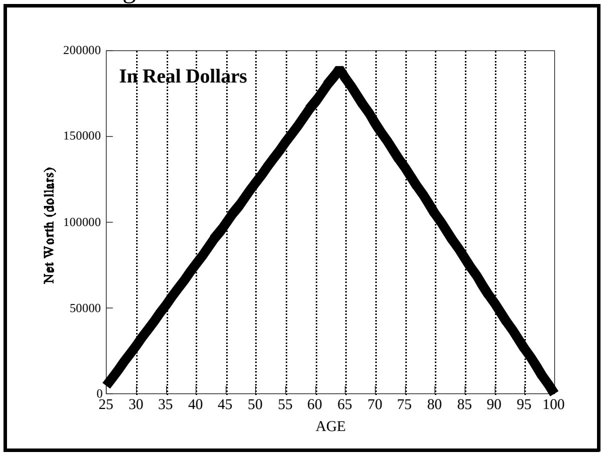


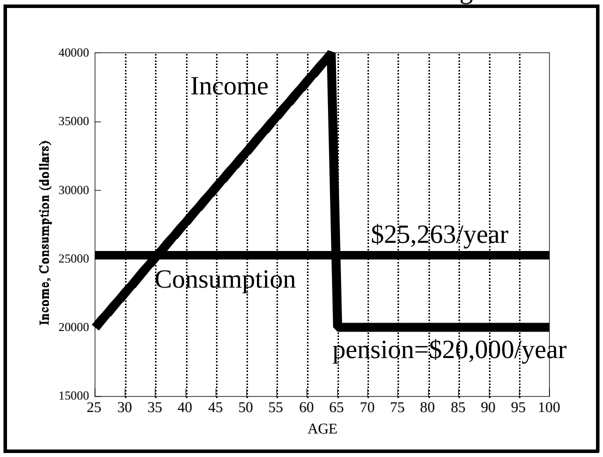
Figure 3.
Scenario 1: Optimal Net Worth from Age 25 to 100, Assuming Constant Real Income of $20,000 Per Year, Then Pension of $10,000 Per Year Until Age 100, Assuming Real Rate of Return of 0% and No Personal Discounting.



Scenario 2: Increasing Real Income

In Figure 4, the household's real income is assumed to increase from $20,000 per year at age 25 to $40,000 at age 64. The earner retires when he/she turns 65. At that point, the pension will be $20,000 per year until death at age 100. The total lifetime income (human wealth) is $1,920,000. The average lifetime income is $1,920,000/76 years = $25,263 per year. According to the simple version of the life cycle model, annual spending should be equal to the average lifetime income.

Figure 4.
Scenario 2: Optimal Consumption Plan from Age 25 to 100, Assuming Real Income Increases from $20,000 Per Year at Age 25 to $40,000 at Age 64, then Pension of $20,000 per Year at Retirement, Assuming Real Rate of Return of 0% and No Personal Discounting.



Each year before retirement, the household should spend $25,263. The first year, income is less than optimal spending, so the household should *dissave*, and $5,263 ($20,000 - $25,263) should be borrowed. The amount borrowed that year is about 26.3% of annual income. The household should keep borrowing until age 36, at which time spending (other than loan repayments) will be less than income (Figure 5.) From age 36 until 64, the percentage of income saved steadily increases, reaching almost 37% at age 64. After retirement, each year, an amount equal to 26.3% of the pension will be withdrawn from the account.

Figure 5.
Scenario 2: Optimal Saving Plan from Age 25 to 100, Assuming Real Income Increases from $20,000 Per Year at Age 25 to $40,000 at Age 64, then Pension of $20,000 per Year at Retirement, Assuming Real Rate of Return of 0% and No Personal Discounting.

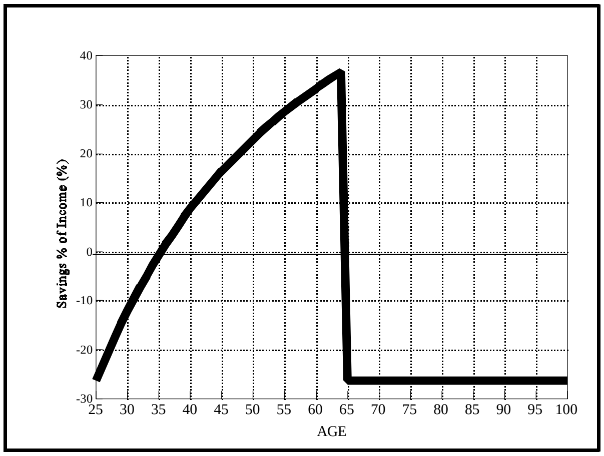
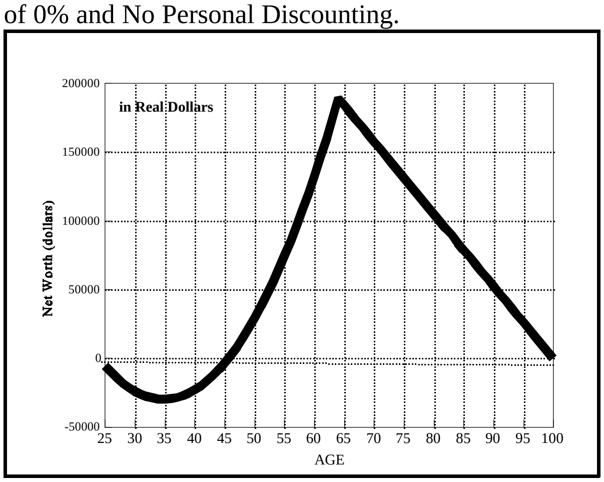


Figure 6.
Scenario 2: Optimal Net Worth from Age 25 to 100, Assuming Real Income Increases from $20,000 Per Year at Age 25 to $40,000 at Age 64, then Pension of $20,000 per Year at Retirement, Assuming Real Rate of Return of 0% and No Personal Discounting.



As Figure 6 shows, net worth first decreases, then increases. The amount *owed* increases to $29,690 at age 35, then the loan balance decreases, reaching zero at age 46. The retirement savings balance increases rapidly, reaching $189,474 at age 64. The savings balance steadily decreases after retirement, reaching zero upon death.

A More Realistic Model

If a consumer must pay interest on loans, borrowing a large amount of money for current consumption becomes much less attractive. It can be shown (Appendix A) that whatever real interest rate a consumer is paying (on loans) or earning after income taxes (on investments) will be an important factor in determining how much real spending (consumption) should grow from one year to the next.

There have been a variety of studies of intertemporal consumption and optimal savings. Appendix B discusses results from selected studies. Important concepts from these studies relate to *risk tolerance*, the *personal discount factor*, the *real interest rate*, and the *liquidity constraint.*

In considering intertemporal decisions, *risk tolerance* relates to how willing one is to tolerate low consumption in one period in order to have much higher consumption in another period. A person with low risk tolerance will attempt to have fairly constant consumption from one year to the next. The *personal discount factor* relates to how impatient one is. In this article, we use the convention that the personal discount factor is related only to objective factors (risk of death and changes in household size.) A consumer will discount next year's consumption by the probability that he or she will be alive then, which for a 20 year old white female is 99.95%, whereas for an 80 year old white male, the discount factor is 91.80% (U.S. Bureau of the Census, 1994, Table 116, p. 88). The cumulative effect over time of discounting for risk of death can be considerable, as the probability that the average 20 year old lives to be 80 is less than 50% (U. S. Center for Health Statistics, 1986).

In order to provide insight into the effect of income patterns on optimal consumption and savings patterns, four simple scenarios are presented. In each scenario, a one person household is assumed, and the personal discount rate is assumed to depend only on the risk of death each year. Perfect certainty is assumed for future real income levels. The consumer makes a plan to cover the entire period from the initial age until age 100. In each scenario, Equation 6 in Appendix A is used to calculate optimal growth in consumption each year. Various levels of initial consumption are tried in an iterative procedure, until the target level of final net worth is achieved.

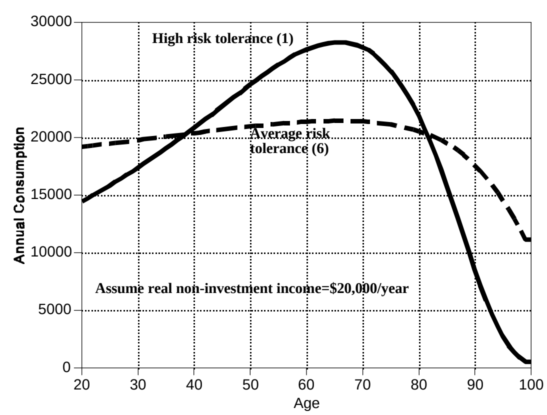
Scenario 3: Constant Income

In Figure 7, real non-investment income after taxes is assumed to be a constant $20,000 per year from age 25 until age 100. Certainty is assumed, all durable goods are leased, there are no bequest motives, and the consumer is fully insured against all possible risks, so that the only motive for saving is life cycle smoothing of consumption. The consumer can invest or borrow at a real interest rate of 2% per year, obviously an unrealistic assumption, but this scenario is intended only to illustrate some implications of the model.

If the risk tolerance level is 1.0, optimal consumption will start at $14,421 per year and increase at approximately 2% per year (Figure 7). The consumer saves 28% of income the first year, but the optimal percent of income to save drops each year, reaching 13% at age 30, and becomes negative after age 38. Real net worth (not shown in Figure 7) increases until age 41, then decreases slowly at first, then more rapidly after age 50. The growth rate of real consumption slows slightly as the annual risk of death increases, until the maximum planned level of real consumption is reached at age 66, at $28,272 that year. Planned consumption (from the point of view of the consumer at age 20) drops rapidly, as the annual probability of death increases. Planned consumption at age 100 is only $556. Although this low amount at age 100 may seem unreasonable, a 20 year old has a very low probability of being alive at age 100, and therefore, (if risk tolerance is 1.0) it would be rational to plan to have very low consumption at age 100.

If the risk tolerance level is 6.0, optimal consumption starts at $19,183 per year and increases at approximately 0.3% per year. The consumer should save 4% of income at age 20, but the percent of income to save will decrease, becoming negative (-0.2%) at age 34 and not becoming positive until age 84. The growth rate of real consumption slows slightly as the annual risk of death increases, until the maximum planned level of real consumption is reached at age 66, at $21,461 that year. The maximum level of net worth is reached at age 35, at a level of $7,258. Planned consumption (from the point of view of the consumer at age 20) drops rapidly, as the annual probability of death increases. Planned consumption at age 100 is only $11,149.

Figure 7.
Scenario 3: Optimal Consumption Plan from Age 20 to 100, Assuming Constant Real Income of $20,000 Per Year, for High Risk Tolerance (1) and Average Risk Tolerance (6), Assuming Real Rate of Return of 2% and Personal Discounting For Risk of Death.



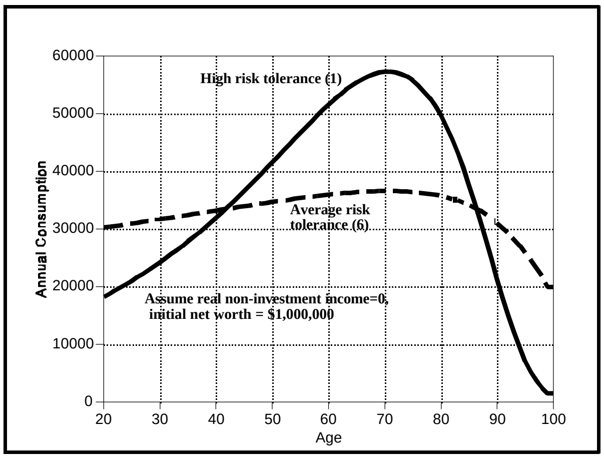
Obviously, the scenario shown in Figure 7 is not realistic. Most people face a drop in real income at retirement, and therefore have to save to prevent a drop in consumption. No person is completely insured against all risks, so precautionary savings are needed. The value of the scenario is to provide insight into a pure consumption smoothing pattern for two values of risk tolerance. It is likely that many consumers would be closer to a level of 6.0 than to a level of 1.0, if faced with the hypothetical scenario.

Scenario 4: Initial Assets of One Million Dollars

In Figure 8, real non-investment income is assumed to be zero from age 20 until age 100. The consumer has safe financial assets at age 20 that yield a real return of 3% per year. As before, certainty is assumed, all durable goods are leased, there are no bequest motives, and the consumer is fully insured against all possible risks, so that the only motive is life cycle smoothing of consumption. This scenario reduces the difficulty of fully understanding the assumption of the first scenario that future income from age 20 until age 100 is known with perfect certainty, as it is only necessary to assume that one could obtain a real rate of return of 3% per year on an investment of one million dollars.

If the risk tolerance level is 1.0, optimal consumption starts at only $18,227 per year (despite the initial assets of $1,000,000) and increase at approximately 3% per year. The growth rate of real consumption slows slightly as the annual risk of death increases, until the maximum planned level of real consumption is reached at age 71, at $57,322. Planned consumption (from the point of view of the consumer at age 20) drops rapidly after age 71, as the annual probability of death increases. Planned consumption at age 100 is only $1,519. Real assets increase until age 43, then decrease to approximately zero at age 100.

Figure 8.
Scenario 4: Optimal Consumption Plan from Age 20 to 100, Assuming Zero Non-Investment Income and Initial Assets of $1,000,000, for High Risk Tolerance (1) and Average Risk Tolerance (6), Assuming Real Rate of Return of 6% and Personal Discounting for Risk of Death.



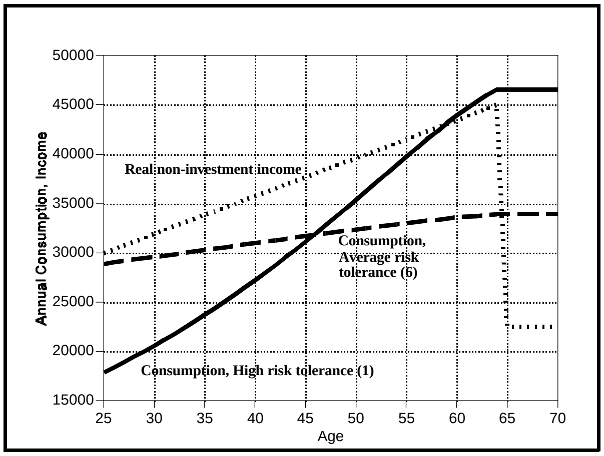
If the risk tolerance level is 6.0, optimal consumption will start at $30,249 per year and increase at approximately 0.5% per year. The growth rate of real consumption slows slightly as the annual risk of death increases, until the maximum planned level of real consumption is reached at age 71, at $36,614 that year. Planned consumption (from the point of view of the consumer at age 20) drops after age 71, as the annual probability of death increases. Planned consumption at age 100 is only $19,987. Real assets steadily decrease from age 20 to age 100.

A remarkable (and counterintuitive) fact about scenario 4 is that planned consumption *increases* even though the consumer starts with a million dollars of wealth, and is certain that he will not receive any additional non-investment income for the rest of his life. This pattern is based on Appendix Equation 7: consumption should increase as long as the real interest rate is greater than the personal discount rate. It seems reasonable to conclude from the extremely low consumption at age 20 for a risk tolerance level of 1.0 that a risk tolerance level of 1.0 is much less plausible than a risk tolerance level of 6.0. However, a risk tolerance level of 1.0 is sometimes assumed (see literature review in Appendix B,) so that level is included in the remaining scenarios.

Scenario 5: Increasing Income

In Figure 9, real non-investment income is assumed to be $30,000 per year at age 25 and increases by $385 each year until age 64, reaching a level of $45,000, then a constant purchasing power pension of $22,500 per year is paid. An arbitrary goal of $200,000 for precautionary savings by age 65 is set, to allow for medical and other emergencies, nursing homes, etc. When the consumer turns 65, most of the accumulated savings is used to purchase a life annuity to supplement the pension, but $200,000 is set aside for emergencies. For simplicity, spending from age 65 on is set at the same level as optimal spending at age 64.

Figure 9.
Scenario 5: Optimal Consumption Plan from Age 25 to 100, Assuming Increasing Non-Investment Income Until Retirement, for High Risk Tolerance (1) and Average Risk Tolerance (6), Assuming Real Rate of Return of 6% and Personal Discounting for Risk of Death.



If the risk tolerance level is 1.0, optimal consumption will start at $17,867 per year at age 25, and the consumer should save 40% of income. The growth rate of real consumption slows slightly as the annual risk of death increases, until the maximum planned level of real consumption is reached at age 64, at $46,576 that year. Assets increase until age 64, reaching $568,302. The percent of non-investment income saved declines steadily, becoming negative after age 58.

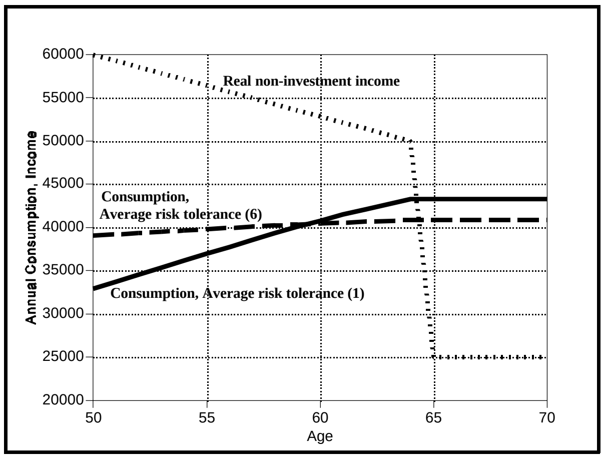
If the risk tolerance level is 6.0, optimal consumption will start at $28,911 per year, with 4% of income saved. The growth rate of real consumption slows slightly as the annual risk of death increases, until the maximum planned level of real consumption is reached at age 64, at $33,917 that year, with 25% of income saved. Assets increase until age 64, reaching $373,612.

The two consumption paths are both optimal, given the assumptions made. The path for the risk tolerance level of 1.0 results in higher net worth than the path based on the level of 6.0, yet neither path is superior, as they cannot be compared in terms of utility.

Scenario 6: Decreasing Income

In Figure 10, real non-investment income is assumed to be $60,000 per year at age 50 and decreases by $714 each year until age 64, reaching a level of $50,000, then a constant purchasing power pension of $25,000 per year is paid. The consumer is assumed to have savings of $100,000 at age 50. An arbitrary goal of $200,000 for precautionary savings by age 65 is set, to allow for medical and other emergencies, nursing homes, etc. When the consumer turns 65, most of the accumulated savings is used to purchase a life annuity to supplement the pension, but $200,000 is set aside for emergencies. For simplicity, spending from age 65 on is set at the same level as optimal spending at age 64.

Figure 10.
Scenario 6: Optimal Consumption Plan from Age 50 to 70, Assuming Decreasing Non-Investment Income Until Retirement, for High Risk Tolerance (1) and Average Risk Tolerance (6), Assuming Real Rate of Return of 3% and Personal Discounting for Risk of Death.



If the risk tolerance level is 1.0, optimal consumption will start at $32,927 per year, with 45% of income saved. The growth rate of real consumption slows slightly as the annual risk of death increases, until the maximum planned level of real consumption is reached at age 64, at $43,299 that year, with 13% of income saved.

If the risk tolerance level is 6.0, optimal consumption will start at $39,063 per year at age 50, with 35% of income saved. The growth rate of real consumption slows slightly as the annual risk of death increases, until the maximum planned level of real consumption is reached at age 64, at $40,887 that year. The percent of non-investment income saved declines steadily, reaching 18% at age 64.

In both cases, planned consumption increases, even though real income declines, and non-investment retirement income is much lower than income at age 50.

Evaluation of Estimates of Risk Tolerance

Based on the examples shown, optimal growth rates of consumption will depend crucially on the interest rate facing each household. The effect of the personal discount rate may also be important. The scenarios presented in this article only consider the possible effect of one observable factor, the risk of death to consumers of different ages. Changes in household size and composition, the health of household members, level of optimism, and other factors may influence the personal discount rate.

Based on these examples, empirical estimates of values of risk tolerance of 1.0 (high tolerance) or less are implausible. Future research is needed to more carefully separate the effect of risk tolerance from the effect of the personal discount rate, and to take into account the income expectations of households. The effects of uncertainty and of savings motivations other than life cycle consumption smoothing also need to be carefully considered.

Implications for Financial Planning

The results of this article should only be considered as providing some useful insights for financial planning, *not* as providing a direct basis for advice to households. One clear implication is that the anticipated pattern of real income in the future is a very crucial part of optimal consumption patterns. Consumers who are sure of substantial increases in real income have less need to start saving a substantial portion of their current income than do consumers who expect constant or declining real income. Scenarios 5 and 6 did include substantial amounts of money set aside for emergencies, specified as an ad hoc withdrawal from life cycle savings at retirement age. This device can provide some flexibility in using the iterative procedure using in this article. It requires additional analysis, however, to determine optimal, or at least plausible levels of emergency funds to accumulate.

Figure 6, based on Scenario 2, does call into question the automatic assumption that increasing net worth should *always* be the goal of a household. Similar patterns can be obtained with more realistic assumptions, as long as the household projects substantial increases in real income (e.g., a doubling of real income between age 25 and 65) and retirement at age 65 or later. Even a non-mathematical analysis of the saving versus spending question should suggest that some households may find it better not to be overly thrifty at particular points of the family life cycle. For instance, in the years just after children are born, family income is often either decreased due to decreased labor force participation, or spending must be increased for child care. The results of the prescriptive life cycle model in effect pose the question, would you be better off with a particular level of consumption when you are young or a higher level of consumption when you are old, at the expense of lower consumption today?

Projection of Real Household Income

Projection of real household income should be based on any planned changes in labor force participation of family members and upon cross-sectional salary patterns. For the latter, it is important to avoid the illusion created by inflation, and make the prudent assumption that the salary level today of experienced workers in one's field will be the purchasing power of one's salary just before retirement. The real increases that can be projected depend a great deal upon the skill level of the occupation. A new assistant professor at the University of Michigan might expect a 60% increase in real salary by the time he or she retired as a full professor (Wright & Dwyer, 1990, 113), whereas a sanitation worker might only expect a 14% increase if he or she retired as a sanitation worker (Wright & Dwyer, 1990, 103). Obviously, not everyone should assume that an income projection is certain. The assistant professor might not make tenure. Or, an assistant professor might hope to become a dean and triple real salary. In general, if real income is projected to double, application of the prescriptive life cycle model based on certainty will result in a recommendation of substantial borrowing for current consumption. If the projection of a doubling of real income is valid, borrowing for current consumption is rational.

If a 25 year old consumer thinks it highly likely that real income will increase substantially, and does not plan to retire until age 65 or later, then it may be rational to defer most saving for retirement until retirement is 25 years away. This may be true even though the consumer would be giving up an opportunity to take advantage of the power of compound interest. Of course, if the consumer should worry about the possibility of being forced to retire earlier than planned, such a delay in starting to save for retirement would not be advisable.

The Effect of the Interest Rate

For a consumer who is rationally considering borrowing for current consumption, the interest rate has an impact on whether to borrow and how much to borrow. However, no particular interest rate is *too high* in general. If the real interest for borrowing is much higher than the real, aftertax rate of return on investments, it may be rational for a consumer to be neither a borrower nor a lender (saver) at some points in the life cycle. The effect on how much is saved each year of having a higher real rate of return on investments (e.g., tax sheltering of retirement saving) may be positive in the short run and negative in the long run, as less investment is needed to accumulate substantial funds for retirement.

Households in the United States face many different interest rates. A household with high levels of financial assets invested conservatively and with high liquidity may face a real, after-tax interest rate of approximately zero. For such a household, the optimal consumption growth may be zero. Households that rent their homes and have negative net worth may face high interest rates, such as the after-inflation rate on a credit card. Some households with poor credit ratings may face interest rates of 30% or more. Therefore, the implications of the life cycle model will be very different for different types of households.

Extensions of the Model

Durable Goods

The analysis presented assumes that all spending is for current consumption, which may be realistic for a consumer who rents a home and leases automobiles. Use of credit for some types of durable goods, such as automobiles, and kitchen/laundry appliances may be rational even if real income is not expected to increase. However, for decisions about how expensive the durable good should be beyond minimum standards (e.g., reliable transportation), the analysis presented in this article may give some insights into how such choices should be made.

Uncertainty

Certainty is assumed in the analyses presented in this paper. Based on simulations of two and three period models with uncertainty, for probabilities greater than 98% that real income will increase or decrease, with the other state of the world being constant real income, there may not be substantial differences in optimal consumption from the certainty cases. In general, the more uncertainty there is about future income prospects, the higher the level of saving (or lower the level of borrowing) there should be (Fan, Chang & Hanna, 1993).

Replanning

The idea that a 25 year old could plan for the next 75 years is unreasonable. However, if the consumer replanned the consumption path each year, adjustments could be made in savings levels. The consumer might have a serious problem if a substantial income drop occurred that was unexpected. For instance, a highly paid steel worker or coal miner who was laid off at age 50 in 1980 might not have anticipated any possibility of such a financial disaster. Prudent financial planning requires consideration of a variety of outcomes, but sudden structural changes in the economy may be difficult to anticipate.

Changes in Household Composition

The analyses presented have been based on a single person household. Consideration of different and changing household compositions will complicate the analysis considerably. It is possible to incorporate equivalence scales into the utility analysis (White, 1978) but more analysis is needed. For a risk tolerance level of 6.0, changes in household size have relatively small impacts on optimal consumption paths, if the equivalence scale for U.S. poverty thresholds are used as part of the personal discount rate.

The Life Cycle Savings Program

A computer program based on the Life Cycle Savings Model is available from Ohio State University (contact first author for more information.) The program can be used to give insight into developing of a financial plan for a particular family and for giving students insights into application of the model to a variety of types of scenarios. The computer program has been used in family resource management and financial planning courses. In the family resource management courses, students have run the computer program with hypothetical case study families. In the financial planning courses, students developing financial plans for actual families have used the life cycle saving program to obtain insights into possible long-run savings plans. For both the hypothetical and actual cases, the projected growth in real income makes a substantial difference in the recommended saving and spending plan by the computer program. If real household income is projected to grow by 2% per year or more for 20 years or more, the computer program typically recommends dissaving. The best approach to this issue is to use pessimistic projections of income. For instance, a new assistant professor might project that real income would increase from now until just before retirement to the level made now by the average associate professor. Pessimistic projections of real income will result in higher recommended amounts of saving each year.

Implications for Financial Management Research

The theoretical model presented in this paper can be used in formulating hypotheses about family financial management behavior and interpreting empirical analyses of financial behavior. For instance, Bae, Hanna and Lindamood (1993) found that, all other things equal, more educated households are more likely to overspend in a year than are less educated households. This result cannot be explained by other models, but it is consistent with the life cycle model. More educated households are more likely than less educated households at the same income level to expect increases in real income. Consideration of the current net asset level of a household, and other factors affecting the real interest rate faced by a household, may also be important in understanding family financial behavior. The basic life cycle model is limited, however, in evaluating situations where uncertainty is important. More complex models (e.g., Fan, Chang & Hanna, 1993,) are needed for uncertainty.

Summary and Conclusion

This article has reviewed the life cycle model and estimates of risk tolerance (relative risk aversion.) Many of the empirical estimates of risk tolerance are not credible, given the intuitive examples presented. The examples presented provide some insight into financial planning issues. For instance, the idea that the same percentage of income should be invested each year is not necessarily valid. A focus on rational real consumption plans can give better insight into plausible savings plans.

Appendix A. The Model

We start with a two period analysis, and then extend it to a life cycle model. Assume that lifetime utility, T, is the sum of the utility from consumption in year 1 and the utility from consumption in year 2:

$$T=U[C_1] + U[C_2] / (1+\rho) \qquad (1)$$

The objective is to maximize T with respect to the following constraints:

$$C_1 = I - S \qquad (2)$$

$$C_2 = (1+g)I + (1+r)S \qquad (3)$$

Variables:
T = Total Lifetime Utility
C_1 = Consumption in Year 1
S = Savings in year 1 (negative value means take loan.)
C_2 = Consumption in year 2
g = Growth rate in income (may be negative)
r = Real interest rate
I = Year 1 Income
Year 2 income = (1+g)I
ρ = personal discount rate. A consumer may discount the future for various reasons, including the possibility of death or disability. The possibility of death provides a simple illustration of personal discounting of the future.

Most studies of intertemporal consumption have used a constant elasticity utility function:

$$U(C_i) = [C_i^{(1+\epsilon)}]/(1+\epsilon) \qquad \{\text{for } \epsilon \neq -1\} \qquad (4a)$$

$$U(C_i) = Ln(C_i) \qquad \{\text{for } \epsilon = -1\} \qquad (4b)$$

The important parameter is the elasticity of marginal utility with respect to consumption, ϵ (White, 1978). By substituting Equations 2, 3 and 4 into Equation 1 and using simple calculus, it can be shown that the optimal ratio of year 2 consumption to year 1 consumption is:

$$\frac{C_2}{C_1} = \left[\frac{(1+r)}{(1+\rho)}\right]^{-\frac{1}{\epsilon}} \qquad (5)$$

If the consumer faces a zero real interest rate, r, then income patterns have no impact on the optimal consumption ratio. With positive interest rates, the optimal consumption pattern will depend on the income pattern, but can be derived from Equation 5. Note that if $\rho = 0$ and $\epsilon = -1$ (corresponding to $U = \ln[C_i]$), optimal consumption should increase by the real interest rate each year. Thus, with a real interest rate of 3%, C_2 should be 3% higher than C_1. *The higher the real interest rate, the higher the optimal growth in consumption*. This is in contrast to Duncan, et al. (1984), who treat an increase in the real interest rate as merely providing the possibility of lower contributions to achieve a given level of living during retirement.

It can be shown that a more general form of Equation 6 will maximize lifetime utility under certain conditions:

$$\frac{C_{n+1}}{C_n} = \left[\frac{(1+r)}{(1+\rho)}\right]^{-\frac{1}{\epsilon}} \qquad (6)$$

For a constant elasticity utility function, it can be shown that this condition will hold for optimal consumption paths with more than two periods, for all pairs of years n and n+1, even if ρ changes from year to year.

The optimal growth rate in consumption can be approximated by the following formula:

$$g_c \approx (r-\varrho)/(-\epsilon) \qquad (7)$$

For reasonable values of the parameters (Appendix B), consumption should grow by the real interest rate divided by 6, or less than 1% per year who have positive net worth, and 2% per year for those who carry balances on credit cards or other high interest credit sources.

Appendix B. Estimates for Risk Tolerance and the Personal Discount Rate

The value of the elasticity of marginal utility with respect to consumption, ϵ, and the value of the personal discount rate ρ, are crucial in determining optimal saving patterns. In the literature, the elasticity of intertemporal substitution in consumption is equal to -1/ϵ. When constant elasticity utility function is used for the analysis of risk, relative risk aversion is -ϵ. In the main body of this article, we refer to -ϵ as *risk tolerance*, and plausible values range from 1.0 (high) to 10.0 (low.) This terminology has the disadvantage of confusing intertemporal decisions (e.g., how much should I consume this year versus next year) with risky decisions (should I invest in stocks or Treasury Bills.) However, in terms of the household's utility function, the effect is similar. With high risk tolerance, as our simulations show, I do not mind very low consumption today in exchange for very high consumption in the future, and I do not mind high consumption in middle age in exchange for the risk of very low consumption at age 80, when I might be dead. With low risk tolerance, I try for fairly constant consumption throughout my life, just as an investor with low risk tolerance is not willing to try for very high returns if there is much risk involved.

For intertemporal consumption, empirical estimates of -ϵ range from -.5 (Shapiro, 1984) to 15 (Hall, 1988). Other estimates were between these two values. Table B.1 contains brief summaries of some of the literature based on empirical estimates. Typically authors have assumed a value for either ϵ or ρ, then estimated the other parameter directly or indirectly. Some authors have treated the personal discount rate separately from discounting based on the risk of death. White (1978) and Skinner (1985) discuss adjustments for changing household size. Although some authors have tested for the existence of a liquidity constraint (Shapiro, 1984), the usual assumption has been that consumers can invest or borrow at the same real interest rate, with 3% as the typical assumed rate. Given that many households have zero or very low levels of liquid assets, empirical estimates based on the assumption of no liquidity constraint are suspect. Allowing for uncertainty in income projections would greatly complicate the analysis. Table B.1 shows the great range of estimates of ϵ and ρ.

The literature suggested a wide range of estimates on the value of the elasticity of marginal utility with respect to consumption, ϵ, and the value of the personal discount rate ρ, and therefore was not very useful in providing guidance for our prescriptive model. Evidence from experiments and surveys imply that the normative theory does not describe consumer behavior very well (Loewenstein & Prelec, 1992) and therefore empirical estimates of these parameters are not meaningful. In order to provide further insight into plausible values of ϵ, an introspective approach is used. These introspective estimates can then be used in the normative (prescriptive) analyses in this article, *assuming* that the anomalies found in research simply reflect the inability of consumers to be rational.

Table B.1
Estimates of Intertemporal Utility Function Parameters from the Literature

ϵ is the Elasticity of Marginal Utility with respect to Consumption. Risk tolerance, as used in this article, equals $-\epsilon$. Relative risk aversion is the same as risk tolerance.

ρ is the Personal Discount Rate.

Davies (1981) Assumed $-5 <= \epsilon <= -.5$. Concluded that the best guess in his simulations is -4.
ρ assumed as the subjective rate of discount, constant over time. In his simulations, $0 <= \rho <= 0.045$; $\rho = 0.015$; LCH is useful in explaining dissaving among the elderly when uncertain lifetime is considered.

Mankiw (1981) ϵ estimated from aggregate data $-6 < \epsilon < -4$. The real interest rate is not assumed to be constant.

Summers (1981) ρ assumed as constant subjective rate of discount, $\rho = 0.03$

Evans (1983) ϵ assumed $-5 < \epsilon < -1$. ρ assumed as subjective rate of discount
ρ: $-0.03 < \rho < 0.03$

Shapiro (1984) ϵ estimated from PSID (Panel Study of Income Dynamics) household data
$\epsilon = -0.5$
The nature of the rejection of LCH suggests the existence of liquidity constraints.

Mankiw, Rotemberg, Summers (1985) ϵ estimated from aggregate data $-4.6 < \epsilon < 0.2$

Skinner (1985) ϵ for individual, is assumed, $\epsilon = -2$
for household, is estimated (by OLS),
$-3.4 < \epsilon < -1.9$ for individual, assumed, z=0.015(time preference rate)
$\rho = (1-z)/m - 1$, where m is the probability of living through next year conditional on living in this year for family unit, ratio of the marginal utility of the surviving spouse is introduced, this ratio is estimated as 0.386.
ρ, Personal discount rate can be computed by using mortality probability data.

Courant, Gramlich & Laitner (1986) ϵ assumed
$\epsilon = -1, -1.5, -2$
ρ estimated as subjective rate of time preference from PSID (1968-1978)
for cross-sectional consumption function, $-0.183 < \rho < 0.012$
for time-series consumption growth function, $-0.113 < \rho < 0.031$
adjusted estimate, $-0.125 < \rho < -0.063$
estimates of ρ are different for people in different age groups, race, education level, and with different consumption functions.
Family size is exogenous in estimating parameters. Uncertainty is not considered.

Von Furstenberg (1988) ϵ Assumed $\epsilon = -1$.
ρ: theoretical analysis of the difference between subjective vs. objective personal discount rate. The smaller the subjective ρ, the higher the non-human wealth for given income.

Hall (1988) ϵ estimated from Livingston Survey
$-15 < \epsilon - 2.9$
estimated from recent monthly data $-3.3 < \epsilon < 3.3$
estimated from Postwar Quarterly Data
$-10 < \epsilon < -2.9$

Attanasio & Weber (1989) ϵ estimated from Family Expenditure Survey (1970-1984)
$\epsilon = -5.1$. Both Expected Utility Approach and Ordinal Certainty Equivalence Approach are used for the estimation.

Hurd (1989) ϵ estimated from. LRHS (1969-1979)
$\epsilon = -.729$ to -1.12
including housing equity $\epsilon = -0.70$ to -1.08 . ρ estimated as constant subjective discount rate $\rho = .0501$ to -0.11
ρ including housing equity =.003 to -.029 (LRHS is Longitudinal Retirement History Survey.)

Zeldes (1989) ϵ estimated from PSID (1968-1982) $-2.7 < \epsilon < -2.3$. Empirical rejections of the permanent

LCH might be due to the existence of liquidity constraint.

Even though it is often assumed that a consumer cannot identify his or her utility function explicitly, it may be possible to construct hypothetical examples that allow one to intuitively identify a unique utility function parameter. Kimball (1988) gives an example to give insight into plausible values of relative risk aversion. It is possible to create a scenario to obtain insight into the similar parameter for the intertemporal utility function. To obtain some insight into plausible values of risk tolerance, consider the following *hypothetical* situation: You are 20 years old, and know with *certainty* that you will live to be 100 in good health. Everything about your personal situation will remain the same for the next 80 years. You want to spend all of your wealth by the day of your death. Your non-asset income will be $20,000 per year in real (constant dollar) terms. You can obtain a return of 6% per year after inflation and taxes on investments. None of the usual reasons for saving exist (e.g., retirement and uncertainty) so that the only reason to save is to take advantage of compound interest with a positive real rate of return. Table B.2 shows optimal consumption paths for different values of risk tolerance. The optimal consumption paths are estimated using a computer simulation program written by the authors, using the model described in Appendix A.

Based on the hypothetical example, a risk tolerance level of 1 (high risk tolerance) would seem extremely miserly, as you would spend only $4,323 of your $20,000 income at age 20 in order to enjoy $457,382 of consumption the last year of your life. A risk tolerance level of 6 might be representative of the typical American consumer, as the consumer would spend $16,929 out of his or her $20,000 income at age 20, and could spend $36,817 at age 100. A risk tolerance level of 20 would have the consumer spending $19,073 out of his or her $20,000 income at age 20, and could spend $24,079 at age 100, thus saving very little out of income. It may seem implausible that someone with high risk tolerance would save more than someone with low risk tolerance, but this result is based on the artificial assumption of no uncertainty about income, investment returns or age at death.

It can be shown that a household should plan to have consumption grow by approximately the real interest rate divided by the risk tolerance level, if the personal discount rate is zero. In other words, if you face a real interest rate of 6% and your risk tolerance level is 6, you should plan your finances so that your real consumption will grow by 1% per year. If the personal discount rate is related to your risk of death, your consumption should grow by approximately 1% per year until age 40, then the growth rate should decrease as the risk of death increases each year.

As the examples in the body of this article show, your expected income pattern has no direct impact on your optimal consumption growth rate, although it does determine your optimal savings pattern.

The *personal discount rate*, ρ might plausibly be related to the chance of death during the year, or to changes in family composition or other factors. For simplicity, the discount factor, $1/(1+\rho)$ is assumed to equal the probability of surviving from year n to year n+1 in this paper. The probability of death used is based on mortality tables for all Americans (U. S. Center for Health Statistics, 1986.) Given the assumption, the value of ρ is approximately the probability of death during the year. The probability of surviving from one year to the next is approximately 99.9% at age 20, so the risk of death will have little influence on year-to-year changes in optimal consumption for those in their 20s. Based on Equation 7, g_c would approximately equal r for $\epsilon=-1$ and r/6 for $\epsilon=-6$ for those aged 20, as ρ would roughly equal 0.00. At age 58, the annual risk of death (and thus ρ) increases to approximately 1% (U.S. Bureau of the Census, 1994, Table 116, 88), and would begin to have a substantial effect on year-to-year changes in optimal consumption. A sixty year old with liquid assets earning a zero or negative rate of return might rationally plan for decreasing consumption. The cumulative effect of such changes after the age of 60 will be substantial for values of risk tolerance near 1.0.

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Table B.2. Optimal Consumption by Risk Tolerance, Hypothetical Example

Age	1 (high risk tolerance)	2	3	4	5	6 ('average')	20 (very low)
20	4,323	11,104	13,942	15,421	16,323	16,929	19,073
30	7,742	14,859	16,931	17,840	18,341	18,656	19,637
40	13,865	19,885	20,560	20,637	20,608	20,558	20,217
50	24,831	26,611	24,968	23,874	23,155	22,655	20,815
60	44,468	35,611	30,320	27,618	26,017	24,966	21,431
70	78,635	47,656	36,820	31,948	29,233	27,512	22,064
80	141,614	63,774	44,713	36,959	32,846	30,318	22,716
90	255,400	85,344	54,299	42,754	36,905	33,410	23,388
100	457,382	114,210	65,939	49,459	41,467	36,817	24,079

