Optimal Credit Use With Uncertain Income

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A two period analysis of optimal credit use for consumers facing uncertain income is developed to give insight into factors related to rational credit use. Extensions of the model to more realistic situations are described.
KEY WORDS: *credit, bankruptcy*

Economic investment theory models developed by Fisher (1930) and Hirshleifer (1970) suggest consumers may increase their utility through judicious selection of debts and assets (Herendeen 1975). A consumer expecting a sustained growth in real income might borrow to smooth consumption over the life cycle. Young consumers, especially students, may find borrowing rational. The availability of consumer credit makes it possible for families and individuals to have immediate consumption of goods and services. However, the dramatic growth of consumer installment debt and use of credit cards during the past two decades (Eastwood 1985, Canner, 1986) has led financial planners and educators to express alarm regarding whether consumers are becoming debt-ridden and overextended. The number of non-business bankruptcies in the United States increased from 312,914 in 1981 to 811,493 in 1991 (Administrative Office of the U.S. Courts, 1991). Clearly there is a need for more understanding of use and overuse of credit.

The purpose of this article is to describe a model of optimal decision-making for credit use with uncertain future income. The analysis is original, but based on theoretical and empirical analysis in the economics literature. Three selected cases are studied. Extensions of the analysis to more realistic situations are described. Although analysis is confined to credit for current consumption, the results may be useful for financial counselors and educators, as well as for insight into empirical patterns of credit use.

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In this article, consumers are assumed to use credit only to "smooth out" current consumption for life cycle purposes. There is no investment in durable goods, saving for retirement, or possibility of loan default. Loans are assumed to be repaid in one lump sum repayment during the second period. Despite these simplifications, this article provides useful original insights into rational credit use. A consumer who is confident that income will increase substantially will rationally take on more credit than will an otherwise similar consumer who does not think that a substantial income increase is likely. Financial educators and counselors should encourage consumers to realistically evaluate the prospects for increases in real income. If an increase in real income is not probable, credit use is not optimal. Hypotheses for empirical studies of credit use should take into account the insights provided by this article.

Review of Literature

There has been extensive discussion in the literature of optimal borrowing-saving and consumption behavior under uncertainty either in the context of infinite time horizon or in the two-period or multiperiod intertemporal models (eg., Leland 1968, Levhari and Srinivasan 1969, Sandmo 1970, Mirman 1971, Dreze and Modigliani 1972, Hey 1979, Sibley 1975). Uncertainty is generally considered in terms of uncertain lifetime, uncertain interest rate, or uncertain income. (For additional discussion of the literature, see Chang, Fan & Hanna, 1992.)

Despite a number of analyses of credit use in the economics literature, there are almost no explicit discussions of economic models in papers with empirical analyses of household credit use. This article provides suggestions for constructing hypotheses for household credit use based on a model of optimal credit use for current consumption.

Description of Method

The two-period model of optimal credit use under uncertain income is based on the assumption that a consumer should maximize expected utility from consumption. The consumer decides how much to spend out of this year's income. Next year's income is uncertain, so the choice of the optimal consumption level this year is not obvious. (The Appendix presents the model rigorously.)

Discussion of Selected Case Studies

Three situations of income increase a consumer might face are analyzed:

(1) **Salary Increase.** There is either a real increase of 1%, e.g., the nominal income growth rate is equal to 6% with an inflation rate of 5% (see Appendix), or real income remains constant, e.g., nominal income increases 5%.;

(2) **Good Promotion.** The consumer changes jobs or gets a good promotion (e.g., is promoted from an assistant professor to associate professor) with a nominal income growth rate of 12% and a real growth rate of 6.7%, or real income remains constant; and

(3) **Additional Job.** For a household, if a member (e.g., wife) is expected to enter or reenter the labor force market in the second year, and a substantial nominal income increase of 50% and a real growth rate of 43% is expected, or real income remains constant.

These three selected cases are studied using the above model with the help of a computer program. Generally speaking, the higher the expected income growth rate, the more the consumer will borrow for a given probability; and the higher the probability of income increase, the more the consumer will borrow for a given real income growth rate. Figure 1 shows the patterns of optimal borrowing-saving for these three cases.

For a nominal income growth rate of 6%, borrowing is never optimal, even if the consumer is certain that the income increase will occur. Note that this analysis does not apply to borrowing for a durable good such as a car. Note

Figure 1
Optimal Saving (Borrowing) as Percent of Year 1 Income, by Probability That Real Income Increases, for Nominal Income Increases of 6%, 12% and 50%, Inflation=5%.

also that some consumers might rationally borrow for current consumption even if they did not expect an increase in income, *if* they did not consider the costs of default very high.

For a nominal income increase of 12%, borrowing is not optimal unless the probability that there will be an increase in real income is at least 39%. If the probability that income will increase is 50%, optimal saving would be -$68 (i.e., optimal borrowing = $68) for a consumer with year 1 income of $20,000, so that the savings/income ratio = -0.3%. If a 12% nominal increase is certain, optimal borrowing would be $411 for a consumer with year 1 income of $20,000, so that the savings/income ratio = -2.1%. For a nominal income increase of 50%, borrowing is not optimal unless the probability that real income increases is at least 15%. If the probability that income increases is 50%, optimal borrowing would be $711 for a consumer

with a year 1 income of $20,000, so that the savings/income ratio = -3.6%. If a 50% nominal increase is certain, optimal borrowing would be $3,757, so that the savings/income ratio = -18.8%.

Discussion

The rate of increase in real income and the probability that real income increases are the most important variables in determining optimal credit use for current consumption. Analyses not presented in this article show that the real interest rate faced by the consumer and the consumer's preferences are much less important in determining optimal credit use.

Durable Goods

The analysis presented assumes that all spending is for current consumption, which may be realistic for a consumer who rents a home and leases automobiles. Use of credit for some types of durable goods, such as automobiles, and kitchen/laundry appliances may be rational even if real income is not expected to increase. However, for decisions about how expensive the durable good should be beyond minimum standards (e.g., reliable transportation), the analysis presented in this article may give some insights into how such choices should be made.

Extensions to More Than Two Periods

If the analysis is extended to three periods, but the assumption is made that the real income level during the third year is whatever the real income level is during the second year, then **optimal credit for the first year is higher than the corresponding level shown in this article for the two period model**. Allowing for changes between year two and year three introduces a much higher degree of complexity, and has not been addressed by the authors of this article.

Allowing for Decreases in Real Income

If there is a possibility that real income will decrease, optimal saving may be positive. For instance, if real income will either remain constant or decrease, and both states of the world are equally likely, the consumer should save some money from year one income in order to prevent too much of a decrease in year 2 consumption. If, however, the probability that real income decreases is small, optimal saving may be approximately zero.

Extensions to More Than Two States of the World

If there are more than two states of the world, analysis of optimal behavior is complex. For consumers with a very small chance of a large decrease in real income, and approximately equal chances of constant real income or a substantial increase in real income, the optimal saving (credit) pattern will be approximately the same as the patterns presented in this article for the two states of the world model. For some consumers, a small possibility of a substantial decrease in income could be dealt with through help from relatives and the social safety net. Consumers could also implicitly assume that decreases in real income could be dealt with by default.

Taking Default into Account

The model used in this article assumes that the possibility of default and/or bankruptcy is ignored. The results described are based on the assumption that the consumer **must** repay the loan in full. There are obvious costs of default and various forms of bankruptcy. If these costs were low, even more consumers would become overextended. It is difficult to specify the monetary value of the costs of default, etc., but given the increasing number of bankruptcies, it is plausible that **a priori**, borrowing even if bankruptcy is possible is rational for some consumers. It is also plausible that many consumers may underestimate the true costs of bankruptcy, and therefore take too much risk with credit use. Most consumers who use credit face at least a small risk of default. The analysis presented in this article provides a starting point to development of a realistic evaluation of rational credit use.

Conclusions

Optimal credit use with uncertain income is described for three scenarios in a two period model. The analysis provides insights into conditions for which credit use may be optimal, although the limitations of the model should be carefully considered before generalization. In a situation where real income can safely be assumed to either remain constant or increase, credit use depends crucially upon the probability that real income increases. This relationship is most evident for a substantial income increase, where the optimal loan is about twice as high for a 100% chance of an increase as for an 80% chance (Figure 1).

Hypotheses for empirical analyses of credit use should take the patterns found in this article into account. The growth rate of real income and the probability that real income increases are the crucial variables in the rational use of credit for current consumption. Researchers designing surveys about credit use should attempt to obtain consumers expectations about income

increases or decreases and the likelihood of increases. Researchers using existing datasets without such information should try to use proxy variables for the income increases and probabilities, such as occupation, age and location.

Financial counselors and educators should encourage consumers to realistically evaluate the chances of income increases. Credit use for investment for durable goods was not considered in this article. For some types of investment in durable goods (e.g., laundry equipment) credit use may be rational even if real income is not expected to increase. Savings goals such as retirement or saving for a down payment for a home were not considered. If a consumer has such goals, credit use is less likely to be rational. Finally, a possible rational basis for consumers becoming overextended can be seen in one of the scenarios. If a consumer thinks that there is a high probability that there will be a substantial increase in income (e.g., $p > 95\%$), then if income does not increase, default and possibly bankruptcy may seem tempting.

Appendix

Equation (1): $\text{Maximize } T = U(C_1) + p*U(C_2) + (1-p)*U(C_{2a})$

The constraints are:

Equation (2): $C_1 = I - S$

Equation (3): $C_2 = (1+g)*I + (1+r)*S$

Equation (4): $C_{2a} = I + (1+r)*S$

Variables:

T = Total two-period utility

I = First period income

C_1 = First period consumption

C_2 = Second period consumption with income increase of g

C_{2a} = Second period consumption without income increase

S = First period saving (Negative S means borrowing)

p = Probability that income increases

g = Growth rate in real income. The real growth rate = (1+nominal growth rate)/(1+inflation rate)- 1.0. Example: nominal growth rate=6% or 0.06. Inflation rate = 5% or 0.05. Real growth rate = (1.06/1.05)-1 or 0.95%. In text, this was rounded to 1%.

r = Real interest rate (which may be different for saving or borrowing) r=(1+nominal interest rate)/(1+inflation rate)-1.

A constant elasticity utility function which is time separable additively is used for intertemporal consumption study:

Equation (5):

$$U = C^{(1-x)} \text{ over } \{(1-x)\}$$

where x is the inverse of the elasticity of intertemporal substitution in consumption. The consumer's relative risk aversion also is x.

To concentrate our study on the relationship between g, p and S, we set x=6 based on extensive literature review and computer simulation (Hanna, 1988; Hanna, Chang & Fan, 1991). The real interest rate r is assumed to be 14.1% for borrowing. This is equivalent to a nominal rate of 19.8% (the typical rate for the largest credit card issuers) if inflation is 5%. It is assumed that the consumer can obtain an interest rate of 1% for saving (e.g., average nominal saving account interest rate of 7.1% with a 5% inflation rate and marginal tax rate of 15%). The results of the analyses are very similar for other plausible values of these parameters.

A closed analytical solution could not be obtained, so simulations were used to find the level of saving (or borrowing) which maximized expected utility.

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