Deterministic and Fuzzy Simulation of Household Financial Planning

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Long-term household financial planning requires flexible, realistic financial planning instruments. This study focuses on the simulation of expenditures for discretionary and nondiscretionary goods and services and the incorporation of uncertainty into a long-term financial planning model. Particular emphasis is given to German financial planning literature and consumer expenditure data sets. Simulation of typical household situations illustrates the effects of family and career planning as well as fuzzy financial data on the amount and crispness of the periodical net surplus. These deterministic and fuzzy simulations are found to be more flexible and exact in predicting household expenditures for different household compositions.

Key words: *Financial planning, financial decision-making, simulation, consumer expenditure survey, fuzzy set theory*

Introduction

Household financial planning usually occurs when people face financial decisions that have far-reaching implications. These financial decisions generally involve a high capital commitment and frequently impact other planning areas of households, such as family planning. In the U.S. as well as in Europe, one of the most important and difficult financial decisions for the average household is financing a home. This is especially true for households whose incomes fall in the two lower quintiles of a nation's net household income. For instance, in Germany, the cost of a home is eight to ten times the average annual net household income (Häußermann and Siebel 1996; Mulder and Wagner 1998). To avoid serious economic problems, housing purchases by families in these income brackets must be prudently planned.

The increasing demand for financial counseling and the increasing number of compulsory auctions of family homes indicates that growing numbers of households are not making wise financial decisions. Financial counselors report that many households still struggle with the first step in financial planning for a home: calculating the surplus funds that can be made available for a purchase of this magnitude (Reifner 1996). Although there is a wide variety of available financial planning instruments, such as workbooks and computer software developed at academic and non-academic settings, opportunities to improve and extend these instruments still exists.

Recent developments in consumer data analysis and simulation methods have made alternative financial planning approaches more useful than traditional financial planning concepts (Zopounidis, Pardalos, and Baourakis 2001; Stryck 1997). The present study shows that the main limitations of financial planning models are the simulation of expenditures for households with changing compositions and the incorporation of uncertain future financial data. This paper proposes to overcome these limitations by incorporating modified equivalence scales and fuzzy set theory into a long-term financial planning model. This new approach blends traditional concepts of long-term financial planning with newer techniques to improve the acceptance and satisfaction of households with the simulation-based planning results.

Limitation of Existing Models

Developing instruments to facilitate household financial planning has been a prominent research area for German household economists. Current financial planning models and methods include instruments for the financial planning of a home (Bertele 1993; Volke 1996), instruments for life-cycle financial planning in farm households (Von Schweitzer 1968; Preuße 1988), an instrument to calculate equivalence scales (Seel 1992), and instruments for everyday household recordkeeping (Preuße and Hagemeier 1995; Warnecke 1997). Despite the academic and practical approval of these instruments, they do not sufficiently address two aspects of financial planning:

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(1) the simulation of expenditures for changing household compositions and (2) the incorporation of uncertainty into a financial simulation. The following two paragraphs introduce these two factors.

Predicting Expenditures for Various Household Compositions

Long-term financial planning covers a planning horizon stretching from five years to several decades. Most households change in size, composition, and standard of living within such a planning horizon (World Health Organization 1976; Höhn 1985; Diekmann and Weick 1993; Clark, Deurloo, and Dieleman 1997). Equivalence scales are the traditional method of measuring these changes and their impact on income and expenditures, including discretionary spending. German household economists developed equivalence scales using the records of selected households (Von Schweitzer 1968; Preuße 1988), continuous household budget surveys of the German Federal Statistical Office (Bertele 1993; Preuße and Hagemeier 1995; Volke 1996), and specially designed household related data collections (Seel 1992). Although the scales reflect the needs of various age groups (i.e. children and adults), economies of scale that result from a growing number of household members have been neglected. Also ignored are the differences in the economies of discretionary versus non-discretionary items. Therefore it seems worthwhile to use two separate equivalence scales. As indicated in Section 2, this dual approach is more flexible and exact in predicting household expenditures for different household compositions.

Incorporating Uncertainty into Long-term Financial Planning

Uncertainty is another concept that is not sufficiently addressed in current long-term financial planning models. Uncertainty plays a dual role in long-term financial planning: the uncertainty of future financial data and the uncertainty of future preferences for allocating financial resources (Möbius 1997). There are some reasons for the existence of these two types of uncertainty. First, households may face significant social and economic changes during a long-term planning horizon. The problems posed by these new situations are increasingly complex and changeable. By implication, households are often not able to determine future financial data and future financial preferences with the required precision (Möbius, 1997, Zimmermann 1993; Mayer et al. 1994). Secondly, they tend to rely on words and subjective reasoning to express present and future financial data. Financial records often are not available (Milling 1982; Mayer et al. 1994; Missler-Behr and Lechner 1996). Thirdly, at the time a long-term plan is created, households may lack sufficient information and rationality to determine both the amount of future financial resources as well as the best way to allocate these resources, as the model of the *homo economicus* might suppose (Milling 1982; Metzendorf 1996).

In developing their financial planning instruments, German household economists have addressed uncertainty by suggesting that households calculate different scenarios using the most probable numbers for income and expenditures (Bertele 1993, Von Schweitzer 1968 and Preuße 1988, Preuße and Hagemeier 1995). A promising alternative to this type of guesswork is the use of fuzzy set theory, which makes allowances for imprecise future financial data and vague resource allocation. Furthermore, the calculus of fuzzy set theory enables the transformation of uncertain financial information into a numerical scale while preserving the imprecision of the information (Koundinya 1995).

In the following sections, both deterministic and more realistic fuzzy versions of a long-term financial planning model are presented. In both versions, the focus is on the long-term simulation of discretionary and nondiscretionary expenditures. This focus was chosen because a household's financial resources depend significantly on a household's willingness to postpone consumption. Discretionary and nondiscretionary expenditures play a major role in this equation.

Model Framework

A major purpose of this study is to improve the long-term simulation of expenditures for discretionary and nondiscretionary goods and services. Households generally have some control over these expenditures, and can expand or reduce them according to the current financial situation. The mathematical terms to calculate expenditures for discretionary and nondiscretionary goods and services are developed in the following paragraphs and used in the simulations in the third and fourth section of this study.

Expenditures for discretionary items can be defined as a function of the household members' age and their participation in the outcome of household production. The expenditures per unit of these goods and services, as well as the amount of these units are highly variable according to household size, composition, and standard of living. Different types of households face different economies of scale for these goods and services.

Stryck (1997) developed the most recent equivalence scales for discretionary spending in German households using the 1988 German Federal Statistics Office's sample survey of income and expenditures (Statistisches Bundesamt 1991). Following a scheme of the German Federal Social Assistant Act (Deutscher Verein für öffentliche und private Fürsorge 1989), Stryck divided discretionary expenses into four categories and developed separate equivalence scales for each group (Table 1). These four categories are (1) food; (2) utilities, fuels, and public services; (3) additional housekeeping needs; and (4) individual needs. Furthermore, to distinguish between the needs of adults and children, the scales include the following age group divisions: Group I—Infancy to age 6; group II—ages 7-12; group III—ages13-17; and group IV—ages 18 and older (including elderly household members). Households with three or more children and economies of scale within one age group are not presented with these equivalence scales (Stryck 1997; Mulder and Wagner 1998). In order to include household with five and more household members and to calculate economies of scale within age groups, the existing equivalence scales are modified in the present study. The modification appears in form of specific allowances for the first three age groups and for the number of household members in one age group (Appendix Tables A-1 to A-4). Due to lack of equivalence scales on the special requirements of the elderly, which would match the equivalence scales developed by Stryck (1997), seniors are considered as regular adults in the simulations.

Equation 1 calculates expenditures for discretionary items, based on the spending practices of a childless couple ($XG_{t,couple}$). This number is multiplied by the parenthetical expression, which calculates the extra expenditures for additional household members in different age groups. In addition, this equation considers the fact that there may be more than one household member in one age group and that there might be additional adult household members. The expenditures are calculated for each of the four categories of discretionary items (Tschoepe 1987; Deutscher Verein für öffentliche und private Fürsorge 1989; Missong and Stryck 1998).

Discretionary goods and services hinder attempts at long-term financial planning because different households anticipate different standards of living in the four discretionary categories. For instance, one household might anticipate a high standard in food quality, a medium standard in personal care products and services, and a low standard in the quality of kitchen appliances. Another household may have completely different expectations. Fortunately, the German "KTBL-Data Collection 'Household'" ("KTBL-Datensammlung 'Haushalt'"; Betz et al. 1991) provides detailed financial ratios for different standards of living. Following the approach of Weinberger-Miller (1989; 1991), a multiplier for three standards of living (low, average, high) for each discretionary category can be aggregated and incorporated into the simulations. To do so, Equation 1 can be extended as shown in Equation 2. In this equation, ($XUG_{bb,t}$) is multiplied by a weighting factor for the standard of living (va_{bb}).

$$XUG_{bb,t} = XG_{t,couple} \cdot \left(1 + \left(a_{i,t} \cdot m_{bb,i,0,t} \right) + \left(\sum_{j=I}^{IV} b_{j,t} \cdot m_{bb,i,j,t} \right) + \left(c_t \cdot m_{bb,0,IV,t} \right) \right) \qquad (1)$$

Equation 1: [$XUG_{bb,t}$ = total expenditures for the particular category of discretionary items in period t; $XG_{t,couple}$ = expenditures of a childless couple for the particular discretionary items category in period *t*; $a_{i,t}$ = 0/1-variable for the firstborn child in the particular age group *i* in period *t*, $a_i = 0, 1$; $m_{bb,i,0,t}$ = addition to the discretionary items category *bb* for the firstborn child in the particular age group *i* in period *t*; $b_{j,t}$ = number of additional children in the particular age group *j* in period *t*, $b_j = 0, 1, ..., B_j$; $m_{bb,i,j,t}$ = addition for an additional child in the particular age group *j* to the particular discretionary items *bb* due to the age group *i* of the first born child in period *t*; c_t = number of additional adults in period *t* (age group IV), $c = 0, 1$; $m_{bb,0,IV,t}$ = addition for an additional adult (age group IV) for the particular discretionary items category *bb* in period *t*; bb = discretionary items category *bb*, *bb* = *n* (food), *s* (utilities, fuels, and public services), *h* (additional housekeeping needs), *p* (individual needs); i = age group of the oldest child, *i* = *I* (0-6 yrs.), *II* (7-12 yrs.), *III* (13-17 yrs.), *IV* (≥ 18 yrs.); j = age group of the succeeding children, *j* = *I* (0-6 yrs.), *II* (7-12 yrs.), *III* (13-17 yrs.), *IV* (≥ 18 yrs.); t = period, *t = 1, 2, ..., T*].

$$XUG_{t,va} = \sum_{1}^{BB} va_{bb} \cdot XUG_{bb,t} \qquad (2)$$

Equation 2: [$XUG_{t,va}$ = sum of expenditures for the particular discretionary items category in period *t* due to standard of living *va*; $XUG_{bb,t}$ = sum of expenditures for the particular discretionary items category in period *t*; va_{bb} = weighting factors for the expenditures due to standard of living; bb = discretionary items category bb, bb = *n* (food), *s* (utilities, fuels, and public services), *h* (additional housekeeping needs), *p* (individual needs); t = period, *t = 1, 2, …, T*].

The present study also examines the nondiscretionary goods and services purchased by households. The long-term simulation of nondiscretionary items differs to some extent from the simulation of discretionary items. Nondiscretionary items are usually not attributable to individuals within a household and therefore the ages of the households have little impact of these expenses (Hertel 1998). Since these goods and services are predominantly independent of the family life cycle phases, they change only incrementally as the number of household members changes (Von Schweitzer 1991). These expenditures are often marked by contractual commitments that determine the amount and the date of the payments (Karg and Volke 1994; Preuße and Hagemeier 1995). Typical examples include rent for a home, insurance premiums, and repayments of loans.

The economies of scale for nondiscretionary items result from the fact that larger households usually allocate fixed costs more efficiently; that is, they take advantage of quantity discounts and of reduced surpluses when purchasing larger quantities of goods. In addition, durable goods are often indivisible and even show characteristics of public goods, such as kitchen appliances or bathroom supplies (Klein 1986; Faik 1995). However, the development of equivalence scales for the expenditures for these goods and services causes statistical difficulties because they are in use over several periods and only replaced irregularly. In addition, substitutional effects between discretionary and nondiscretionary items cause analytical problems (Stryck 1997). As a matter of fact, no specific equivalence scales are available which would allow predicting the expenditures for nondiscretionary items during changing household compositions.

Table 1
Equivalence Scales for Expenditures for Discretionary Goods and Services

Household Type	Food	Utilities Fuels, Public Services	Add'l. House-keeping Needs	Indivi-dual Needs
Single person	1.060	1.426	1.384	1.678
Couple without children	2.000	2.000	2.000	2.000
Couple with one child, 0 to 6	2.090	2.144	2.332	2.192
Couple with one child, 7 to 12	2.404	2.240	2.240	2.228
Couple with one child, 13 to 17	2.724	2.296	2.296	2.106
Couple with two children, 0 to 6	2.310	2.278	2.428	2.194
Couple with two children, 0 to 6 and 7 to 12	2.552	2.344	2.466	2.320
Couple with two children, 7 to 12	2.730	2.730	2.414	2.230
Couple with two children, 0 to 6 and 13 to 17	2.730	2.536	2.558	2.476
Couple with two children, 7 to 12 and 13 to 17	3.022	2.480	2.482	2.160
Couple with two children, 13 to 17	3.156	1.255	1.285	2.324

Source: Stryck 1997

In the present study a pragmatic solution was found in terms of the so called "welfare scale" of the German Federal Social Assistance Act (Brühl 1999). This equivalence scale is based on income assistance to welfare households (Appendix Table A-5). It assigns the weight 1.00 unit to the reference person. Each additional adult household member is assigned the weight 0.80; the weights for children vary according to their ages, from 0.50 (ages 0 to 6 years), 0.65 (ages 7 to 12) and 0.90 (ages 13 to 17) (Deutscher Verein für öffentliche und private Fürsorge 1989; Hauser 1996; Brühl 1999). Like the equivalence scales proposed in the present study to predict expenditures for discretionary items, the "welfare scale" has only four age groups and does not consider the number of household members in the same age group (Klein 1984). Again, for the simulations in the following section, the "welfare scale" has been modified adding allowances for different age groups and thus improving the calculations of economies of scale for nondiscretionary goods and services (Appendix Table A-6).

It is important to note that the "welfare scale" mirrors not only the empirically calculated relations of minimum consumption, as is the case in the equivalence scale of the discretionary items, but also normative political decisions. This explains the higher weights for children compared to other empirically developed scales. For instance, the OECD weights every household member, besides the household head, with 0.7 units and each child under 15 years with 0.5 units (Krämer 2000). For further details on equivalence scales see, for instance, Burkhauser (1994), Merz and Faik (1995), or Hauser (1996). Although a financial plan ought to be as realistic as possible, households should be conservative in their assumptions; therefore, the financial reserve funds created by using the "welfare scale" seem to be reasonable.

The simulation of expenditures for nondiscretionary items using a particular household size and composition (XUW_t) will be calculated in the simulations of the next section as shown in Equation 3. The reference groups are the expenditures of a childless couple ($XW_{t,couple}$), which are multiplied by the allowances for additional household members, presented by the parenthetical terms.

$$XUW_t = XW_{t,couple} \cdot \left(1 + \left(\sum_{i=I}^{IV} a_{k,t} \cdot m_k\right)\right) \quad (3)$$

Equation 3: [XUW_t = sum of expenditures for nondiscretionary goods and services in period *t;* $XW_{t,couple}$ = expenditures of a childless couple for nondiscretionary goods and services in period *t;* $a_{i,t}$ = expenditures for the additional household members in the particular age group *i* in period *t*, $a_{i,t} = 0, 1, ..., A_{i,t}$; m_i = addition for every additional household member in the age group *i*, $m_i = 0.50; 0.65; 0.90; 0.80; 5/9$ (for single persons); i = age group of the additional household member, *i = I* (0-6 yrs.), *II* (7-12 yrs.), *III* (13-17 yrs.), *IV* (≥ 18 yrs.); t = period, $t = 1, 2, ..., T$].

When calculating nondiscretionary expenditures over a long-term planning horizon, additional attention should be turned to durable consumer goods. They represent the most expensive nondiscretionary items and, therefore are more costly to purchase, repair, and replace (Münnich and Illgen 1999). These expenditures are usually calculated by the depreciation method. In the simulations of the present study, linear depreciation, which several authors consider valid for most practical applications (Blosser-Reisen 1991; Karg and Lehmann 1991), has been used to predict replacement and repair costs for nondiscretionary items. However, the problem of reporting and valuating the durable goods is considerable, since type and diversity of durable goods in households vary widely (Claupein 1990). In order to solve this problem, the simulations in the present study included only so-called indicator goods— household appliances, such as furniture and fixtures; electric appliances; television, radio, sound equipment; computer, telephone, fax machines; and vehicles. This approach is consistent with the German Federal Statistics Office's sample survey of income and expenditures (Münnich and Illgen 1999). Equation 4 calculates the expenditures for repairs and replacement of indicator goods (gt) in each period ($XUWA_t$). The depreciation sum and the anticipated lifespan of the goods are presented in Tables 2 and 3.

$$XUWA_t = \sum_{gt=1}^{GT} \frac{AS_{gt,t}}{(i_{gt,t} - n_{gt})} \quad (4)$$

Equation 4: [$XUWA_t$ = sum of expenditures for repairs and replacement of the durable goods in period *t;* $AS_{gt,t}$ = depreciation sum of the good *gt* in period *t;* $i_{gt,t}$ = time period of depreciation of the good *gt* in period *t;* n_{gt} = anticipated life of consumer good *gt* with $(i - n) \geq 0;$ gt = type of durable consumer good (indicator), $gt = 1, 2, ..., GT;$ t = period, $t = 1, 2, ..., T$].

Table 2
Depreciation Horizon for Nondiscretionary Items in EUR/year

	Household Type		
Expenditure categories	Couple without children	Couple with one child	Couple with two children
Furniture and fixtures	20	16	13
Electric appliances	15	12	10
Television, radio, sound equipment	15	12	10
Computer, telephone, fax machine	7	6	5
Vehicles	15	12	10

Source: calculations based on Betz et al. (1991)

Table 3
Depreciation Sums for Nondiscretionary Items in EUR/year

	Household Type		
Standard of living	Couple without children	Couple with one child	Couple with two children
Middle	-2.843	-3.532	-4.271
Low	-1.421	-1.766	-2.135
High	-5.686	-7.065	-8.542

The depreciation sums are calculated expenditures. In the simulations below they are collected in an intermediate "pool" account. The repair and replacement costs of durable goods do not affect cash flow until they exceed the amount of money in the "pool" account.

To summarize, a framework for long-term financial planning has been proposed that offers substantial progress in the long-term simulation of discretionary and nondiscretionary items. The advances include: refined equations for predicting expenditures for both discretionary goods and services (Equation 1); different standards of living (Equation 2); nondiscretionary goods and services (Equation 3); and periodical depreciation of durable goods (Equation 4). In the next section this framework is used to simulate two household scenarios.

Deterministic Simulation

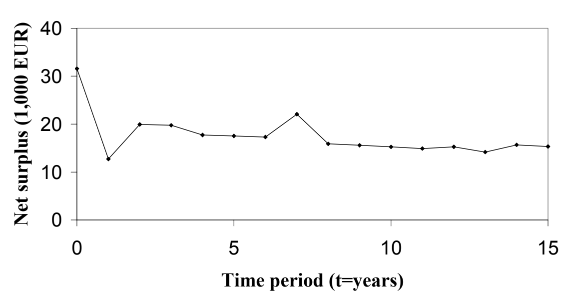
To illustrate the proposed financial planning model, two household scenarios are simulated. The simulations are used to calculate the periodical financial net surplus of a household, which is the sum of its income and expenditures in one period. Equations 1 to 4 and the related equivalence scales are embedded in the calculation of the net surplus. To emphasize the different cash flow in households, income and expenditures are bundled mutually exclusively into three major groups: (1) earned and unearned income and related expenditures; (2) maintenance income and expenditures, including the expenditures for discretionary and nondiscretionary items as well as repayment of loans, savings, income from the sale of used or home-produced goods; and (3) transfer income and expenditures, which represent payments by governments, businesses, and households to households and vice versa. Unlike the other components of a household's income and expenditures, which represent cash flow for services rendered, transfers are characterized as income to and expenditures for households for which they have not rendered current services (Transfer-Enquete-Kommission 1981). Transfer expenditures cover such items as taxes, insurance premiums, membership fees, or cash contributions. Donations, heritages, and grants count as transfer income.

The financial data for the first period of the planning horizon for each of the three groups are noted in the Appendix Tables A-7 to A-11. These data are the mean income and expenditures of German households according to the data provided by the 1993 German Federal Statistics Office's sample survey of income and expenditures (Statistisches Bundesamt 1997a; b; c; Braun 2000). The periodical growth rate of income and expenditures is calculated according to the time series of the Statistic Yearbook for the Federal Republic of Germany (Statistisches Bundesamt 2000). To keep the presentation fairly concise, it is further assumed that each household has an average standard of living relative to its expenditures for discretionary items (cf. Equation 2) and uses durable goods for an average period of time (cf. Equation 4). In addition, it is assumed that the standard of living remains constant during the planning horizon, thus ensuring that the overall period net surplus is not calculated at the expense of a household's standard of living. The planning horizon covers 15 *years* because the failure of home financing most often occurs within this period (Reifner 1996). The simulations were processed with the software *JBuilder 6,* available from Borland Software Corporation, Scotts Valley, CA 95066-3249.

The scenarios simulate the most common type of German household: couples who purchase a home during the first year of marriage and plan to give birth to one or two children within the first decade of marriage (Ulbrich 1997). The first scenario represents a couple with one child, born already in the first year of marriage and a planned parental leave of only one period. The second scenario presents a couple with two children, whose births are delayed to periods 5 and 7, combined with a prolonged parental leave.

It is assumed that the first couple plans to have a child at period 1 after the purchase of a home (t = 0). It is further assumed that the parent who takes leave following the child's birth will return to half-time employment at period 2 and full-time employment at period 7. The average annual net surplus for this scenario is illustrated in Figure 1.

Figure 1
Net surplus of a couple with one child household

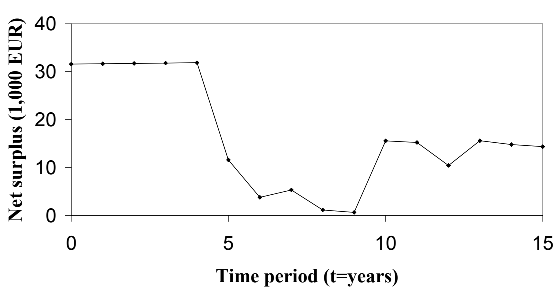


Family planning: first child at t=1;
Career planning: working half time in t=2, full time in t=7

Over the 15-year planning horizon, this household shows an average net surplus of 17,534 EUR/year (1 EUR = 1.19 USD in 1993; Euro replaced Deutsche Mark in 2001: 1 EUR = 1.96 DM), which is based on 67,875 EUR/year for earned and unearned income minus 59,022 EUR/year for maintenance income and expenditures plus 8,681 EUR/year for transfer income and expenditures. Complete results of the simulations in this study are available from the author upon request. Figure 1 illustrates that the net surplus decreases significantly in the first period because of the stay-at-home status of one parent and the additional expenditures for the new family member. After the stay-home parent returns to half-time employment (t = 2), the family's financial situation improves only to when they lose their government-subsidized benefits, which expire after three years of support (t = 4). If both the standard of living and expenditures for durable consumer goods remain constant, the financial situation improves when both parents work full time (t = 7). Two, full-time incomes help to cover the increasing costs of discretionary and nondiscretionary items for a growing child (t = 8). A slight improvement in the financial situation is achieved when the child moves to the third age group (t = 14).

Figure 2 presents a second typical household scenario, the delayed birth of the first child combined with a prolonged parental leave. It is assumed that the couple plans to give birth to children at periods 5 and 7. It is further assumed that the parent who takes leave following the child's birth returns to half-time employment at period 10 and full-time employment at period 13, when the children are ready for daycare and school.

Figure 2
Net surplus of a couple with two children household



Family planning: first child at t=5, second child at t=7;
Career planning: working half time in t=10, full time in t=13

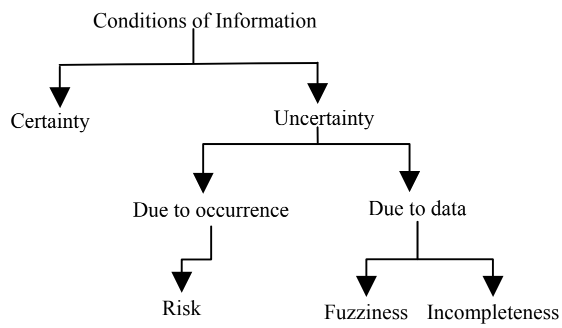
The simulation shows an average overall net surplus of 16,687 EUR/year, which is based on 66,189 EUR/year earned and unearned income, minus 58,617 EUR/year maintenance income and expenditures, plus 9,115 EUR/year transfer income and expenditures. The interrupted employment of one parent diminishes the net surplus during periods 5 to 9. The family benefits, which the German government provides during these periods, do not completely replace the missing wages. Assuming that the standard of living and the expenditures for durable consumer goods remain constant during the planning horizon, the overall net surplus during periods 8 and 9 is very small. To cover the cost of purchasing a home, the household needs to consider either reducing its maintenance expenditures, liquidating assets, or borrowing extra capital during these periods. Additional half-time employment in period 10 and full-time employment by period 13 helps to increase the household's net surplus as well as its ability to handle increasing expenditures for discretionary and nondiscretionary items required by a growing household.

Although only the first three progressive stages of the family life cycle (Word Health Organization 1976) have been simulated to keep the presentation concise, these two scenarios emphasize the importance of life planning in calculating the future financial situation of a household. The date of the first child's birth and the date that the stay-home parent returns to employment are key factors. Obviously, two incomes better balance the rising costs of a growing household. The longer that a couple delays starting a family, the greater the likelihood that they will have an adequate reserve to cover major financial obligations, the costs of having a second child, and a longer stay-home period for one parent. If the stay-home parent resumes employment shortly after the child's birth, the additional expenses of the new household member can be made up with greater ease. The earlier that a parent starts working again after the birth of the child(ren), the better the financial situation. These findings also underscore the importance of current and elastic equations and of equivalence scales to simulate expenditures during changing household structures.

Fuzzy Arithmetic Simulation

For the simulations presented in the last section, a household is expected to indicate exact numbers for future expenditures and to have precise plans for allocating financial resources during the planning horizon. Often, households cannot provide exact financial data, although many have a good idea of what they expect to earn and spend during the planning horizon. A purpose of this study is to incorporate vague, fuzzy data into a financial planning model to make it more useful. Figure 3 illustrates the different types of uncertainty prevalent in financial planning.

Figure 3
Conditions of Information



Source: Brunner (1994); Hönerloh (1997)

Although fuzzy uncertainly plays a major role in long-term financial planning, it is largely overlooked in the literature. Fuzziness is characterized by vague information, which results from the innately qualitative, fuzzy character of future financial data (Buscher and Roland 1992). Fuzzy set theory is a method that incorporates imprecise information in mathematical structures. By contrast, probability theory deals with uncertainty related to precise but incomplete information (Koundinya 1995; Müller 1998). Contrary to fuzzy set theory, probability theory assumes that there exists a collection of possible outcomes, concerning period of occurrence or amount of payment, for future financial data. By implication, it is possible to define probabilities of events within the financial plan and to employ stochastic simulation or the Monte-Carlo Method. For a detailed discussion of probability theory vs. fuzzy set theory see Kruse, Gebhardt, and Klawonn (1993) or Dubois and Prade (2000).

Zadeh (1965) introduced the concept of fuzzy sets using the set- and relation-definitions of modern algebra (Böhme 1993; Bothe 1993; Klir, St. Clair, and Yuan 1997). Much has been written about fuzzy sets in the past two decades (Koundinya 1995); in addition, the concept has been applied to economic thought, see for example Klir and Folger (1988), Tilli (1993), Zimmermann (1996) or Pedrycz and Gomide (1998). However, applying fuzzy set theory to financial decision-making in micro-economics is still in its infancy (Koundinya 1995) and has not yet reached household economics. The present study makes a unique contribution to financial planning literature— it is the first study to incorporate methods developed within the fuzzy set theory into a long-term financial planning instrument for households.

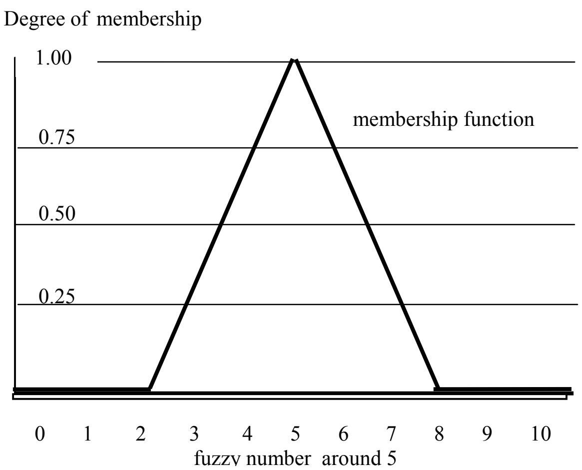
Fuzzy Numbers

To apply fuzzy set theory to long-term financial planning, it is important to understand the concept of "fuzzy numbers." A fuzzy number is a number word and a linguistic modifier, such as approximately, nearly, or around (Klir, St. Clair, and Yuan 1997). For instance, the expression "approximately five" is fuzzy because it includes some number values on either side of its central value of five, i.e. the values 2, 3, 4 and 6, 7, 8. The central value "5" is fully compatible with the expression; the numbers around the central value are compatible with it to lesser degrees (Klir, St. Clair, and Yuan 1997). Altogether, these numbers represent a "fuzzy number." The concept is useful in financial planning for households who cannot characterize future income and expenses in crisp numbers.

Every fuzzy number is defined by a mathematical function, the so-called membership function. Membership functions are the core of the analytic framework of fuzzy set theory (Bellman and Zadeh 1970; Rommelfanger 1994; Hönerloh 1997). To create a membership function for a fuzzy number, the function must represent a set of numbers around a given real number (Klir, St. Clair, and Yuan 1997). It is a convention that the membership function of fuzzy numbers assigns fuzzy levels to the number, which range from zero (if the uncertainty cannot be bordered at all) to one (if the number is met optimally). For instance, the degree of 1 would be assigned to the central value, i.e. "5", and degrees between zero and one to the surrounding values that reflect their proximity to the central value, i.e. the degree of 0 to the value 2 and smaller ones, the degree of 0.3 to the value 3, the degree of 0.6 to the value 4, etc. (Böhme 1993; Mayer et al. 1994; Rommelfanger 1994).

The determination of membership functions for fuzzy numbers in long-term financial planning is subjective and should be based on detailed knowledge of the financial situation in households (Koundinya 1995). Fuzzy numbers can be represented by membership functions of various shapes; the most common ones are linear (Böhme 1993; Kruse, Gebhardt, and Klawonn 1993; Mayer et al. 1994; Hönerloh 1997; Möbius 1997; Müller 1998). Isosceles triangles represent the simplest linear shape for membership functions and according to several authors, they are valid for most practical applications (Zimmermann 1978; Rommelfanger 1994; Möbius 1997; Müller 1998). In addition, they represent an adequate compromise for the simulations presented below because they combine an adequate formal effort with a reasonable demand for financial information to the user, the planning household. Figure 4 illustrates the fuzzy number "5," including the real number 5, both limiting value "2" and "8", the degrees of membership between 0 and 1, and the triangular-shaped membership function.

Figure 4
Fuzzy number 5



The shape of the membership function is not the only factor that defines the fuzziness of financial data. If a household can determine membership degrees for fuzzy financial data, levels can be defined, which narrow the range of a fuzzy number. These levels can be viewed as crisp sets within fuzzy numbers (Klir, St. Clair, and Yuan 1997). A method to restrict membership degrees is particularly important for financial planning simulations; these are the α-cuts (Mayer et al. 1994; Rommelfanger 1994; Zimmermann 1996; Hönerloh 1997). An α-cut of a fuzzy number is the crisp set that contains the entire set of numbers of a fuzzy number whose membership degrees are greater than or equal to the specified value of $\alpha \in [0, 1]$ (Kruse, Gebhardt, and Klawonn 1993). For example in Figure 4, an α-cut of $\alpha = 0.5$ would be illustrated by a horizontal line stretched parallel to the abscissa and cutting the ordinate in the value 0.5. The reason that great importance is placed on the α-cut*s* of fuzzy numbers is that any fuzzy number may be completely characterized by its α-cuts (Klir, St. Clair, and Yuan 1997). Thus, the α-cuts present an instrument to reduce or enlarge the vagueness of future financial data according to the subjective reasoning of a household.

Once membership functions and α-cut levels are determined subjectively or otherwise, arithmetic operations on fuzzy numbers can be defined. Since each fuzzy number is uniquely represented by its α-cuts and these are closed intervals of real numbers, arithmetic operations with fuzzy numbers can be performed using the rules of classical interval analysis (Klir, St. Clair, and Yuan 1997). For more detailed insights on transferring interval analysis to fuzzy set theory, see Bonarini and Bontempi (1994). One way to formulate the four basic arithmetic operations on arbitrary fuzzy numbers is to represent the numbers by their three main parameters, the central value, i.e. *m* or *n*, and the limiting values, i.e. α, β or γ, δ. To explain how this is done, arbitrary fuzzy numbers $\tilde{A}_1 = [m; \alpha; \beta]$ and $\tilde{A}_2 = [n; \gamma; \delta]$ are considered as shown in Table 4.

Table 4
Basic arithmetic operations on fuzzy numbers

	Interval analysis
$\tilde{A}_1 + \tilde{A}_2 =$	$[m + n; \alpha + \gamma; \beta + \delta]; \forall \tilde{A}_1; \tilde{A}_2$
$\tilde{A}_1 - \tilde{A}_2 =$	$[m - n; \alpha - \delta; \beta - \gamma]; \forall \tilde{A}_1; \tilde{A}_2$
$t \cdot \tilde{A}_1 =$	$t \cdot [m; \alpha; \beta] = [tm; t\alpha; t\beta]; \forall t \geq 0$
	$t \cdot [m; \alpha; \beta] = [tm; -t\alpha; -t\beta]; \forall t < 0$

Source: Dubois and Prade (1980); Hönerloh (1997)

The advantage of these algorithms lies in their simplicity as well as in their flexibility in expressing the vagueness of data. Furthermore, they can be easily programmed to facilitate the integration of fuzzy numbers in a financial planning simulation. For this paper, the arithmetic operations of addition, subtraction and scalar multiplication are sufficient. Approximation equations for fuzzy multiplication and fuzzy division, which preserve the triangular shape of the given fuzzy numbers, are based on simplified algorithms (Dubois and Prade 1980; Böhme 1993; Mayer et al. 1994). For a detailed derivation of these algorithms, see Böhme (1993). In the simulations of the next paragraph, the concept of fuzzy numbers is implemented in the deterministic model. This approach is called "fuzzy arithmetic simulation."

Steps in Fuzzy Arithmetic Simulation

Fuzzy arithmetic simulation blends traditional, deterministic simulation with the concept of fuzzy numbers. Again, the simulations are focused on long-term expenditures for discretionary and nondiscretionary goods and services. But in addition, it is assumed that these expenditures are fuzzy. All other income and expenditures are held crisp to keep the present study within reasonable limits.

Fuzzy arithmetic simulation is a two-step process (Hönerloh 1997). In the first step, the upper and lower boundaries of the overall net surplus are calculated on the basis of anticipated highest and lowest expenditures for discretionary and nondiscretionary items. At the same time, all other financial data are the same as in the deterministic simulation (Appendix Tables A-1 to A-5). This first step is not only a technical procedure but should also help to sensitize households to financial extremes during changing family structures. The second step includes the simulation on the basis of α-cuts of the fuzzy numbers. To this end, membership functions and α-cut levels for the fuzzy numbers are integrated in the simulations.

Figure 5 illustrates the first step of the fuzzy arithmetic simulation process. The simulation data are the same as in the deterministic simulations except for the expenditures for discretionary and nondiscretionary items. For the purpose of demonstrating how fuzzy set theory works, it is assumed that the financial data for the discretionary and nondiscretionary items ranges 25 percent above and below the mean financial data. This range defines the limits of the fuzzy numbers. The simulations were run for the household scenario that included a couple with one child, comparable to Figure 1.

When discretionary and nondiscretionary expenses are underestimated by 25 percent, the simulations result in an average net surplus of 32,289 EUR/year Compared to the mean net surplus in Figure 1, the average annual net surplus is increased by 14,756 EUR/year or 54 percent, which would allow the family to purchase a more expensive property. However, this advantage is based on lower discretionary and nondiscretionary spending over the entire planning period. The contrary situation is presented when discretionary and nondiscretionary expenditures are overestimated. The average net surplus is lowered to 2,778 EUR/year. The financial situation is obviously precarious. The assumed high expenditures for discretionary and nondiscretionary items cause at least seven shortfall periods with annual deficits.

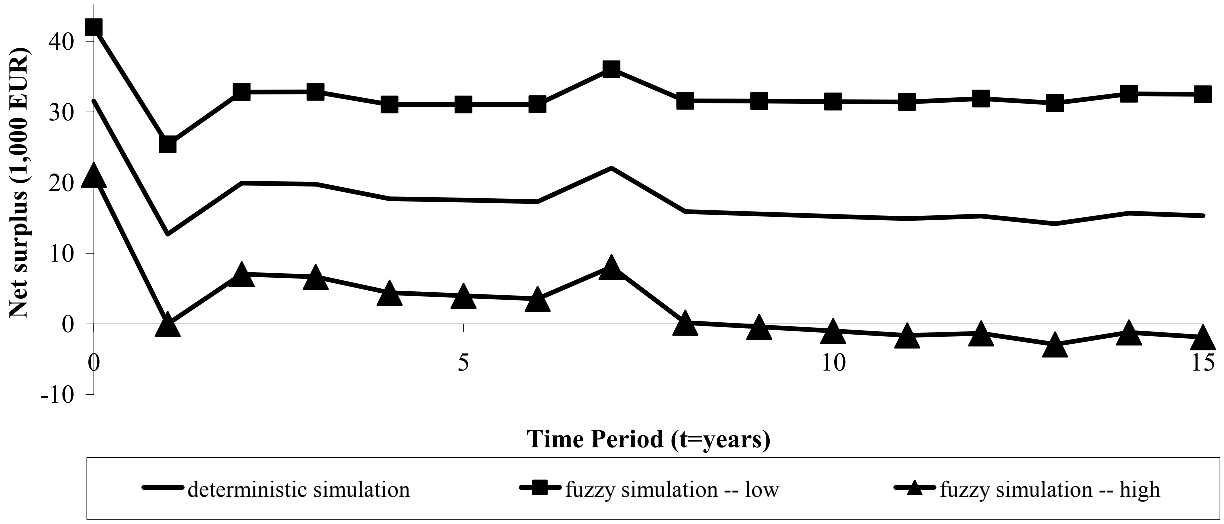
A compromise solution between both extreme values can be developed with α-cuts. This is the second step of the fuzzy arithmetic simulation. Λ-cut levels provide the opportunity to estimate fuzzy numbers individually and narrow them according to the subjective reasoning of the planning household (Bonarini and Bontempi 1994; Klir, St. Clair, and Yuan 1997). When applying α-cut levels of $\alpha = 0.5$ and $\alpha = 0.3$ to the "couple with one child" scenario, the results differ significantly from the "first-step" results for the net surplus/deficit. At the α-cut level of $\alpha = 0.5$ this household disposes of an average net surplus of 10,156 to 24,911 EUR/year. The range averages ± 7,378 EUR/year. The compromise solution represents a financial planning program with a structure that resembles the extreme and deterministic solutions, but lies in-between these solutions. This is illustrated in Figure 6.

Taken together the results of the two-step fuzzy arithmetic simulation, in combination with α-level sets, allow uncertain financial data to be incorporated into a financial planning model for households. Especially with the definition of α-cut levels, a planning household can express the possible range of each fuzzy number. In a real case simulation, this method allows a household to vary ranges and α-cut levels for each financial data, if desired. In so doing, a household becomes aware of the degree of uncertainty in the planning process.

On the other hand, the simulations confirm the limitations of this method. Specifically, the intervals of the fuzzy numbers broaden with each additional algebraic operation. By implication, the interpretation of the fuzzy numbers becomes more difficult as more algebraic operations are undertaken. Solutions are the ongoing "de-fuzzy-fication" of fuzzy numbers and "feeding" the next simulation step with crisp numbers, ongoing interpretation of (intermediate) results, or some combination with scenario techniques.

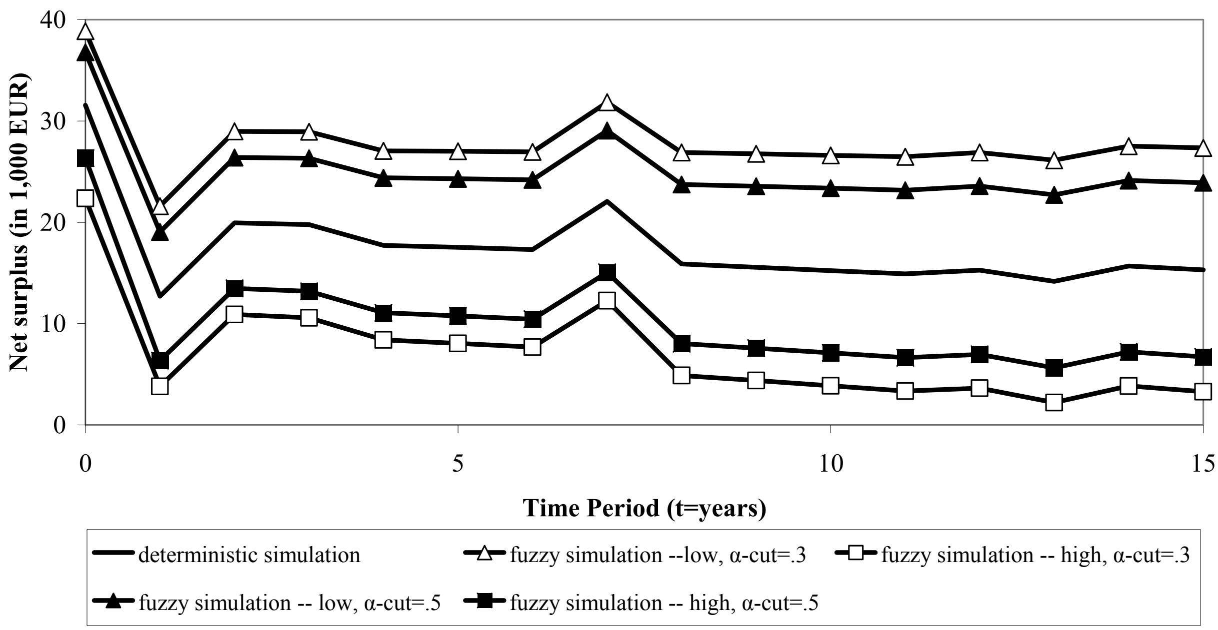
Regardless of strategy, it is worth emphasizing that the objective of a simulation that incorporates fuzziness is to model the financing planning with the exactness that appears in the household's future financial data. Thus, an exact crisp net surplus may not be the primary goal of the simulation.

Figure 5
Deterministic and fuzzy simulation results for extreme low and high maintenance



Fuzzy simulation--low assumes extremely low maintenance;
fuzzy simulation--high assumes extremely high maintenance
Family planning--first child at t=1; Career planning--working half time in t=2, full time in t=7

Figure 6.
Deterministic and fuzzy simulation results with α-cuts of .3 and .5



Fuzzy simulation--low assumes extremely low maintenance;
fuzzy simulation--high assumes extremely high maintenance
Family planning--first child at t=1; Career planning--working half time in t=2, full time in t=7

Conclusions and Implications

Responsible financial planning adapts to each individual household situation. Both family and career planning play important roles in determining the eventual net surplus or deficit. Therefore, it is helpful to outline the important stages of the life plan of each household member. The earlier and more frequently this happens, the more successful long-term financial planning will be. Although no planning instrument can guarantee the stability of the anticipated net surplus at the time of planning, long-term financial planning pursues worthwhile financial goals. First, the interdependences between income and expenditures and household-endogenous factors, such as size, composition, and standard of living of a household, can be calculated. Second, the financial planning model provides transparency about the financial resources of a particular household. A household becomes informed about the net surplus available for financing or investing and becomes sensitized to elements of uncertainty. Finally, while delivering input data for the simulation, a household is forced to deal with its financial determinants and to express them concretely.

In addition, there are a number of advantages to applying the fuzzy set theory. First, it integrates uncertainty into a unique decision-making process by incorporating the household's level of uncertainty in future financial data into the formal mathematical model via membership functions. In addition, α-cut levels enable households to adjust the fuzziness of every future financial data according to subjective reasoning. Another advantage of the exposed methodology is that it considers the possible instability of the household's preferences, which is consistent with evidence from several household economic studies (Seel 1975; Blosser-Reisen 1980; Galler and Ott 1993). Households are prepared, to a certain extent, to abandon savings objectives in favor of other consumption interests or because the allocation of their financial resources is too vaguely defined. Fuzzy arithmetic simulation deals more suitably with these variables. It requires ongoing analysis, however, to observe the evolution of fuzzy set theory and to adjust it for valid simulation of socio-economic systems.

Three extensions of the model are envisioned. First, the implementation of time decision-making could be considered. In the study presented here, the variable "time use of the household members" is assumed as exogenously given. Variations are restricted to different dates for the birth of children and a window of time for paternity/maternity leave. A variable linking the time dimension with finances would provide households the possibility to reconsider the allocation of family and working hours. In general, the more pressing a household's need to increase the net surplus, the shorter a period for paternity/ maternity leave would be planned. A second extension of the model could focus on intra-household decision-making. In the presented scenarios, a household is regarded as an economic unit. Intra-household decision-making in terms of the household member's different characters and financial intentions has been neglected (Lipsey 1971; Galler and Ott 1993; Haddad, Hoddinott, and Alderman 1994). It would be useful to explore the motivations that lead to particular financial decisions.

A third extension of the presented financial planning instrument could incorporate the idea of "re-planning.". For expect a household to plan for the next 15 years is quite unreasonable; therefore, a household should periodically check its family and career situation. If circumstances change, additional simulations may be required to arrive at financial decisions that better reflect the situation. This approach is known as "rolling planning" (Adam 1996; Möbius 1997).

This study introduces refined equivalence scales and presents a first contribution to fuzzy set theory in long-term financial planning in households. These two innovative approaches have potentially important implications concerning the reliability of financial planning instruments. They offer potential for a serious analysis of a household's present and future financial situation to have a meaningful impact on economic decision-making, particularly in households that tend to underestimate the consequences of long-term financial obligations.

Appendix

Table A-1
Modifications of the equivalence scale for food expenditures

Allowance for the oldest child in the age group			Allowance for the younger siblings in the age group I		II		III	
I	+0.090	$(m_{n,I,0})$	+0.220	$(m_{n,I,I})$	-		-	
II	+0.404	$(m_{n,II,0})$	+0.152	$(m_{n,II,I})$	+0.326	$(m_{n,II,II})$	-	
III	+0.724	$(m_{n,III,0})$	+0.006	$(m_{n,III,I})$	+0.298	$(m_{n,III,II})$	+0.432	$(m_{n,III,III})$

Table A-2
Modifications of the equivalence scale for expenditures for utilities, fuels, and public services

Allowance for the oldest child in the age group			Allowance for the younger siblings in the age group I		II		III	
I	+0.144	$(m_{s,I,0})$	+0.134	$(m_{s,I,I})$	-		-	
II	+0.240	$(m_{s,II,0})$	+0.104	$(m_{s,II,I})$	+0.130	$(m_{s,II,II})$	-	
III	+0.296	$(m_{s,III,0})$	+0.240	$(m_{s,III,I})$	+0.184	$(m_{s,III,II})$	+0.214	$(m_{s,III,III})$

Table A-3
Modifications of the equivalence scale for expenditures for additional housekeeping needs

Allowance for the oldest child in the age group			Allowance for the younger siblings in the age group I		II		III	
I	+0.332	$(m_{h,I,0})$	+0.096	$(m_{h,I,I})$	-		-	
II	+0.240	$(m_{h,II,0})$	+0.226	$(m_{h,II,I})$	+0.174	$(m_{h,II,II})$	-	
III	+0.296	$(m_{h,III,0})$	+0.262	$(m_{h,III,I})$	+0.118	$(m_{h,III,II})$	+0.274	$(m_{h,III,III})$

Table A-4
Modifications of the equivalence scale for expenditures for individual needs

Allowance for the oldest child in the age group			Allowance for the younger siblings in the age group I		II		III	
I	+0.192	$(m_{p,I,0})$	+0.002	$(m_{p,I,I})$	-		-	
II	+0.448	$(m_{p,II,0})$	-0.128	$(m_{p,II,I})$	-0.218	$(m_{p,II,II})$	-	
III	+0.106	$(m_{p,III,0})$	+0.370	$(m_{p,III,I})$	+0.054	$(m_{p,III,II})$	+0.218	$(m_{p,III,III})$

Table A-5
"Welfare Scale" for different household types

Household Type	weights in 1999
Single person	1.111
Couple without children	2.000
Couple with one child	
Couple , one child, age 0 to 6	2.556
Couple , one child, age 7 to 12	2.722
Couple , one child, age 13 to 17	3.000
Couple , one child, age 18 and older	2.889
Couple with two children	
Couple , two children, ages 0 to 6	3.111
Couple , two children, ages 0 to 6 and 7 to 12	3.278
Couple , two children, ages 7 to 12	3.444
Couple , two children, ages 0 to 6 and 13 to 17	3.556
Couple , two children, ages 7 to 12 and 13 to 17	3.722
Couple , two children, ages 13 to 17	4.000
Couple , two children, ages 0 to 6 and 18 plus	3.444
Couple , two children, ages 7 to 12 and 18 plus	3.611
Couple , two children, ages 13 to 17 and 18 plus	3.889
Couple , two children, ages 18 plus	3.778

Table A-6
Modifications of the equivalence scale for expenditures for nondiscretionary items

Allowance for the oldest child in the age group			Allowance for the younger siblings in the age group I		II		III	
I	+0.556	(m_0)	+0.556	(m_I)	-		-	
II	+0.722	(m_0)	+0.556	(m_I)	+0.722	(m_{II})	-	
III	+1.000	(m_0)	+0.556	(m_I)	+0.722	(m_{II})	+1.000	(m_{III})

Table A-7
Simulation data for the first period for earned and unearned income and in EUR/year

	Household Type		
Types of income	Couple without Children	Couple with one Child	Couple with two Children
Net earned income[a]	54.204	46.273	49.437
Sale of tangible and monetary assets	8.846	7.947	8.007
Interests, dividends, rental income, other property income	5.658	5.437	6.066
Consumer credit	276	549	667
Other earned and unearned income	542	462	494

Source: calculations based on Braun (2000), *Statistisches Bundesamt* (1997b)
[a] Income reduced by employment related expenses

Table A-8
Simulation data for first period for expenditures for discretionary items in EUR/year

Types of expenditures	Household Type Couple without Children
Food	-3.768
Utilities, fuels, and public services	-1.350
Additional housekeeping needs	-2.227
Individuals needs	-12.376

Source: calculation based on *Statistisches Bundesamt* (1997b)

Table A-9
Simulation data for the first period for the expenditures for nondiscretionary items in EUR/year

Types of expenditures	Household Type Couple without Children
Furniture and fixtures	17.000
Electric appliances	5.350
Television, radio, sound equipment	5.250
Computer, telephone, fax machine	1.800
Vehicles	14.125

Source: calculations based on 2000 consumer prices

Table A-10
Simulation data for the first period for additional maintenance income and expenditures in EUR/year

	Household Type		
Types of payments	Couple without Children	Couple with one Child	Couple with two Children
Repayments of Loans Plus Interest Payments	-558	-887	-1.037
Expenditures to Acquire Tangible Assets	-6.381	-8.386	-8.861
Expenditures to Acquire Monetary Assets	-9.369	-9.274	-9.580
Expenditures for Maintenance and Repair of the Property	-811	-932	-966
Income from the Sale of Used or Home Produced Goods	372	591	480

Source: calculations based on *Statistisches Bundesamt* (1997a)

Table A-11
Simulation data for the first period for transfer income and expenditures in EUR/year

	Household Type		
Types of payments [a]	Couple without Children	Couple with one Child	Couple with two Children
Taxes (without Income Tax)	-632	-611	-611
Premiums to and from personal insurance (without Social Security)	-1.424	-1.605	-1.647
Membership Fees and Cash Contributions	5.826	7.429	6.782
Social Security; tax reductions; public assistance, Supplemental Security Income; Unemployment Compensation	0[b]	13.378	18.716

[a] Income Tax and Health Insurance payments are not included in the subgroups; in Germany, these payments are subtracted by the employer and therefore not included in the net household income.

[b] The flow of transfer income and expenditures has been simplified to emphasize the intergenerational transfer payments to families with children, which is characteristic for young families buying a home.

Source: calculations based on Braun (2000), *Statistisches Bundesamt* (1997b)

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