Breakeven Periods For Individual Retirement Accounts With Partial Withdrawals

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If the money invested through an IRA is withdrawn before the investor turns 59½, it is subject to a penalty of 10%. Therefore, it is unclear whether an IRA is preferred to an ordinary investment or not. If the money remains invested in an IRA for a sufficient time, the IRA breaks even despite the penalty. In this paper, an expression is derived for the break-even period for a partially withdrawn. It is shown that the break-even periods are shorter than previously thought. Some simple rules of thumb are proposed for circumstances when the IRA will be attractive and circumstances when investors should stay away from the IRA.

KEY WORDS: *individual retirement accounts, emergency funds, income-tax, retirement planning, investment*

Introduction

The United States Internal Revenue Service (IRS) may allow a tax-payer to deduct a contribution to that tax-payer's Individual Retirement Account (IRA) from gross income to arrive at the taxable income. Whether the contribution can be deducted or not depends upon the magnitude of each taxpayer's income and whether the taxpayer or spouse is covered under a retirement plan.[a] Such a deduction lowers the tax-payer's immediate tax liability; additionally, the returns on any amount invested through the IRA are not taxable until withdrawals are made. At that time, the withdrawn amount is taxed at the tax-payer/investor's then prevailing tax rate. Thus, the two benefits of the IRA are that taxes on contributions are deferred and the interest on investments is tax-free. However, if money is withdrawn before the investor is 59½ years old, it is subject to a 10% penalty.

Kelly and Miles (1987) show, under an assumption of no penalties, if the investor's tax rate is constant from the time that the contribution is made till closure of the account then the IRA always has a positive Net Present Value (NPV), i.e., it is unequivocally beneficial to invest through the IRA. However, if the funds in the IRA are withdrawn when the withdrawals are subject to a penalty, it is not clear whether the IRA is a worthwhile investment or not because the investor might have been better off with an ordinary investment which is free from any IRS imposed constraints or penalties. Under some circumstances it is possible, with the tax-free compounding and tax deferral on the contribution, that the IRA is better than the ordinary investment even if the withdrawals are penalized.

Specifically, the IRA is worthwhile if the money remains invested at least for the *break-even* period. The break-even period is the time at which the after-taxes-and-penalty value of the IRA is equal to the value of the ordinary investment. Collins (1980) shows that this break-even period varies with the assumptions about the tax rate during the investor's working periods and at the time of withdrawal. Smith (1982), Bogan and Bogan (1982), and Smith (1984) derive expressions for the break-even period on an IRA where the fully accumulated amount is withdrawn and penalized and calculate it for various rates of return while holding the tax rate constant throughout the investor's life. Clute and Reichenstein (1989) show that even the non-deductible IRA can be an attractive alternative for intermediate term investments because its break-even period is short when returns are relatively high.

One of the assumptions commonly made in the literature on break-even periods is that all the accumulated money in the IRA is withdrawn. Since the investor makes the choice of how to much to withdraw, the assumption that the IRA is fully withdrawn is unrealistic. The purpose of

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this paper is to derive an expression for the break-even period for a partially withdrawn IRA. Calculating the break-even period on a partially withdrawn IRA is more realistic because it accounts for the timing and magnitude of withdrawals, which are both under the investor's control. Another advantage of this approach is it recognizes that at the time of withdrawal a question any investor might ask is how much more time must the funds remain invested in the IRA for it to break even. The approach that is adopted in this paper allows this question to be answered, while the previous analyses ignore it altogether because they assume that the IRA is fully withdrawn.

The comparative-statics of the break-even period are also derived. Under appropriate conditions, it can be shown that the break-even period decreases when the tax rate increases, when the interest-rate rises, and when the withdrawal is deferred, while it increases with an increase in the proportion withdrawn. The break-even period is calculated for a sample of tax rates, interest rates, portion of IRA withdrawn, and time of withdrawal. It is shown that the average break-even period is shorter than the estimates published in the academic and practitioner literature. Further, some rules of thumb are proposed on the circumstances under which the IRA is attractive and the circumstances under which investors should stay away from the IRA.

An Example

The model for the break-even period on a partially withdrawn IRA that is proposed here is presented in the Appendix in order to focus upon the results and implications of the model. For an example that uses equation A8, suppose an investor in the 28% tax bracket contributes $1,500 to an IRA at time zero. The investment earns interest at a rate of 6% compounded continuously. At the end of the second year, this investor is compelled to withdraw enough cash from the IRA to meet a $400 emergency. The remaining amount continues to earn interest till maturity which is after the investor reaches age 59½. As shown in Panel A of Table 1, the IRA grows to $1,691 by the end of the second year when the investor withdraws $645 from the IRA before penalties and taxes to meet the $400 emergency. The remaining $1,046 grows to $1,172 by year 3.89, which is the break-even period according to the formula with α equal to 38.15%. At that time, the full IRA is subject to taxes at a rate of 28%. After paying taxes, the investor is left with a sum of $844.

Had an ordinary investment been opted for instead, the investor could not have deducted the $1,500 IRA contribution from gross income. Therefore, the investor would have forfeited the tax benefit of that contribution. Since the 28% tax rate renders a tax benefit of $420, the equivalent investment in the ordinary account is $1,080 ($1,500 less $420). A calculation, similar to the IRA, for the ordinary investment is shown in Panel B of Table 1. The ordinary investment grows to only $1,177 at the after-tax interest rate of 4.32% when the emergency withdrawal of $400 is made. The remaining $777 grows to an expected $844 by year 3.89.

In this example, the break-even period of 3.89 years is short despite more than one-third of the IRA funds being withdrawn at an early stage, especially once it is recognized that the break-even period occurs only 1.89 years after the withdrawal. How does the break-even period respond to changes in the tax rate, the proportion withdrawn, the interest-rate, and the time of withdrawal? To answer that question the comparative-statics of n^* are derived from equation A8. Unfortunately, the signs of $\frac{\partial n^*}{\partial t}$, $\frac{\partial n^*}{\partial \alpha}$, $\frac{\partial n^*}{\partial r}$, and $\frac{\partial n^*}{\partial j}$ are indeterminate in general.[b] However, some interesting statements can be made about the partials when some restrictions are placed upon the parameters under which the partials are calculated. First and foremost, the partials are evaluated only when $(n^*-j) > 0$ because when $(n^*-j) \leq 0$, it means that the break-even period has been reached by the time of withdrawal; once that occurs it is irrelevant how the break-even period responds to changes in the parameters. Second, p is restricted to 10%. This is consistent with the existing treatment of early withdrawals. Third, t is restricted to 15%, 28%, 31%, 36%, or 39.6% because these rates conform to the existing progressive tax structure for personal income taxes in the U.S. Finally, r is restricted to values from 4% to 22%. The justification for selecting these particular interest rates is provided in the next section.

It appears that

$$\frac{\partial n^*}{\partial t} < 0, \frac{\partial n^*}{\partial \alpha} > 0, \frac{\partial n^*}{\partial r} < 0, \text{ and } \frac{\partial n^*}{\partial j} < 0 \quad \text{whenever } (n^*-j) > 0.$$

[c] The break-even period falls when the tax rate rises, or when the interest rate rises, or when the withdrawal is deferred, and it rises when the proportion withdrawn increases. These results are interesting and important from a financial planning perspective because they provide answers to the following sorts of questions:

(1) What happens to the desirability of an IRA if the interest rate that can be earned rises? (Since the partial of n^* with respect to the interest rate is negative, the IRA becomes more desirable because the break-even period falls when the interest rate rises). (2) Does it make sense for a tax-payer to invest in an IRA if his income is expected to increase and subsequently he is likely to pay taxes at a higher marginal rate? (Yes, because the IRA becomes even more attractive since the partial derivative of n^* with respect to taxes is negative which means that the break-even period falls as the tax rate increases). (3) Does the IRA become more desirable if the withdrawal is made sooner than later? (No, the IRA becomes less desirable because the partial of n^* with respect to the time of withdrawal is negative). (4) What happens to the desirability of the IRA if a larger rather than a smaller amount needs to be withdrawn? (The IRA becomes less attractive because more money will be lost to the penalty and this is borne out by the positive sign on the partial of n^* with respect to the proportion withdrawn).

Table 1
IRA Break-even period Example *

Panel A: The IRA			
Amount contributed to an IRA	A	=	\$1,500
Future value of the IRA at the time of emergency (time 2) (Eqn. A1)	$Ae^{rt} = 1500e^{(.06)(2)}$	=	\$1,691
Pre-tax amount that has to be withdrawn to meet the emergency	$\frac{E}{(1-p-t)} = \frac{400}{(1-.10-.28)}$	=	\$645
The proportion of the IRA that has to be withdrawn to meet the emergency (Eqn. A2)	$\alpha = \frac{\$645.16}{\$1,691.25}$	=	38%
Amount remaining in the IRA after withdrawal	\$1,691.25 - \$645.16	=	\$1,046
Future value of the remaining amount at the break-even period (3.89 years)**	$\$1,046.08e^{(.06)(1.89)}$	=	\$1,172
Future value after taxes are paid at a rate of 28%	$\$1,171.63(1-.28)$	=	\$844
Panel B: The Ordinary Investment			
After-tax contribution to the ordinary account	$A(1-t) = \$1,500(1-.28)$	=	\$1,080
Future value of the ordinary investment at time 2	$A(1-t)e^{r(1-t)j} = \$1,080e^{(.06)(.72)(2)}$	=	\$1,177
Amount to be withdrawn to meet the emergency	E	=	\$400
Amount remaining after withdrawal	\$1,177.46 - \$400	=	\$777
Future value of the remaining amount at the break-even period	$\$777e^{(.0432)(1.89)}$	=	\$844

*Investor is in the 28% tax bracket throughout, and the interest rate is 6% compounded continuously. At time 2, an emergency occurs with a net (after-tax) requirement of \$400. The notation that is used above is defined in the Appendix.

**The break-even period comes from Eqn. A8 with t=.28, r=.06, α=.38, p=.10, and j=2, i.e.,

$$n^* = \frac{\ln\left(\frac{[(1-t)-\alpha(1-p-t)e^{rjt}]}{(1-\alpha)(1-t)}\right)}{rt} = \frac{\ln\left(\frac{[(1-.28)-.38(1-.10-.28)e^{(.06)(2)(.28)}]}{(1-.38)(1-.28)}\right)}{(.06)(.28)} = 3.89$$

Results

In this section, the break-even period n^* is calculated under various scenarios with respect to t, α, r, and j. Results are reported for tax rates of 15% and 28% only because it is unlikely that investors in higher brackets would be able to deduct their contribution under the existing tax code.[d] Since the number of possible scenarios is infinite, break-even periods are reported for interest rates ranging from 4% to 22% in increments of 6% points, and α of 25%, 50%, and 75%. The selected interest rates are typical for these types of studies. For instance, Smith (1982) calculates the break-even period for interest rates from 6% to 30%, Bogan and Bogan (1982) for interest rates from 4% to 15%, Smith (1984) for interest rates from 6% to 16% and Clute and Reichenstein (1989) from 6% to 21%. Therefore, the range is consistent with the received literature. To ensure that results are reported for a wide range of proportion withdrawn, α has been selected to range from 25% to 75%.

Table 2
The relative break-even period.
This table shows the number of years that an investor has to remain invested in an IRA after the partial withdrawal has been made to break even. A zero in any cell means that the IRA has reached the break-even period. Each panel contains results for a different proportion withdrawn.

Panel A: Twenty-five percent of the accumulated amount is withdrawn

	Tax rate 15%				Tax rate 28%			
Time of	*Interest rate*				*Interest rate*			
withdrawal	4%	10%	16%	22%	4%	10%	16%	22%
1	5.13	1.28	0.32	0.00	2.76	0.34	0.00	0.00
2	3.84	0.00	0.00	0.00	1.48	0.00	0.00	0.00
3	2.55	0.00	0.00	0.00	0.20	0.00	0.00	0.00
4	1.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Panel B: Fifty percent of the accumulated amount is withdrawn

	Tax rate 15%				Tax rate 28%			
Time of	*Interest rate*				*Interest rate (r)*			
withdrawal	4%	10%	16%	22%	4%	10%	16%	22%
1	16.74	5.61	2.83	1.56	9.85	2.87	1.12	0.31
2	14.94	3.79	0.98	0.00	8.07	1.05	0.00	0.00
3	13.13	1.95	0.00	0.00	6.27	0.00	0.00	0.00
4	11.31	0.08	0.00	0.00	4.46	0.00	0.00	0.00
5	9.48	0.00	0.00	0.00	2.64	0.00	0.00	0.00
6	7.64	0.00	0.00	0.00	0.79	0.00	0.00	0.00
7	5.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	3.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	2.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Panel C: Seventy-five percent of the accumulated amount is withdrawn

	Tax rate 15%				Tax rate 28%			
Time of	*Interest rate*				*Interest rate*			
withdrawal	4%	10%	16%	22%	4%	10%	16%	22%
1	47.41	17.15	9.57	6.10	28.25	9.54	4.83	2.66
2	44.40	14.06	6.38	2.82	25.33	6.48	1.59	0.00
3	41.35	10.86	3.02	0.00	22.35	3.22	0.00	0.00
4	38.26	7.56	0.00	0.00	19.31	0.00	0.00	0.00
5	35.14	4.13	0.00	0.00	16.19	0.00	0.00	0.00
6	31.98	0.58	0.00	0.00	13.00	0.00	0.00	0.00
7	28.77	0.00	0.00	0.00	9.72	0.00	0.00	0.00
8	25.52	0.00	0.00	0.00	6.36	0.00	0.00	0.00
9	22.23	0.00	0.00	0.00	2.91	0.00	0.00	0.00
10	18.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
11	15.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00
12	12.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
13	8.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00
14	5.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	1.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The calculated break-even periods are reported in Table 2. The table is divided into panels A, B, and C, one for each α. The first four columns of each panel contain the break-even periods for investors in the 15% tax bracket while the last four contain results for the 28% tax bracket. For example, to find the results for an investor in the 28% tax bracket who withdraws 75% of his IRA, look in the last four columns of Panel C of the table.

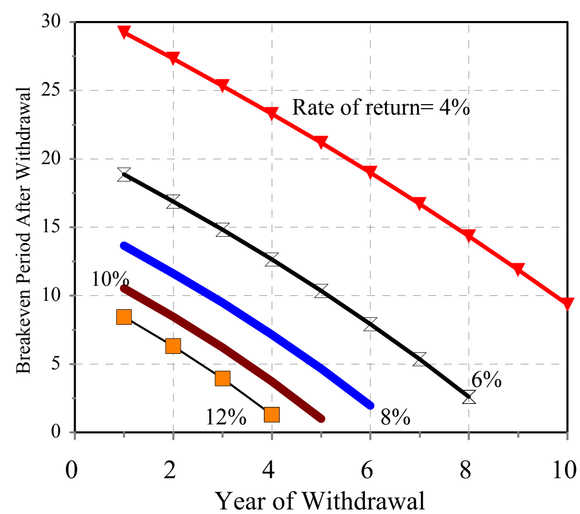
In calculating the break-even periods, j is assumed to be an integer for the sake of convenience. At the time of withdrawal, most investors would be concerned with how much more time they must remain invested in the IRA to be no worse-off than investing in an ordinary account. Therefore, "relative" n^* is reported, i.e., (n^*-j) is shown. If at the time of withdrawal the IRA has reached the break-even period, i.e., (n^*-j) #0, a "0.00" appears in that cell. Once (n^*-j) is zero for a particular j for all interest rates, no further results are reported for that α because, as shown above, n^* decreases as j increases.

For example, consider the following scenario: Suppose an investor who pays taxes at a marginal rate of 15% withdraws 50% of the accumulated money in the IRA at the end of the 3rd year. Assuming that the IRA has been growing at an interest rate of 10%, how long will it take for this investor to be no worse off relative to an ordinary investment? From panel B of Table 2, it appears that it will take another 1.95 years for this investor's IRA to break even. Clearly, the particulars of this example were contrived to fit the parameters for which results have been provided in Table 2. The formula in equation 8 can be easily used for financial planning purposes by plugging in the relevant numbers to calculate the break-even period.

As another example, consider Figure 1. For a 75% withdrawal, for someone in a 28% marginal tax rate, the break-even period decreases as the rate of return increases from 4% to 12% in increments of 2 percentage points (the break-even periods for interest-rates of 6%, 8%, and 12% are not shown in Table 2; however, they have been calculated in the same way). Furthermore, for any particular rate of return, the break-even period decreases as the time of withdrawal increases.

There are three important issues that are addressed in the discussion of the results. First, for investors who are seriously considering the IRA as an investment vehicle, the uppermost question in their minds would be under what circumstances should they invest through the IRA without fear of being worse-off relative to an ordinary investment? To answer this question, the circumstances under which the IRA breaks even soon after withdrawal are pointed out. Second, under what circumstances should they absolutely stay away from the IRA? To answer this question, the general circumstances under which it is virtually impossible to salvage the IRA are highlighted. If any of these circumstances are applicable to particular investors, they should stay away from the IRA. Third, how do the break-even periods compare to the ones reported previously in the literature?

Figure 1.
Example of Breakeven Periods for 75% Withdrawal, by Time of Withdrawal, for Five Rates of Return, 28% marginal tax rate.



By looking at the results that are presented in Table 2 certain regularities are apparent. In particular, short break-even periods share certain characteristics. First and foremost—and this is no surprise—if only a small proportion (25% or less) of the IRA is withdrawn then chances are that the IRA will break even within approximately 5 years because all cells, with one exception, in Panel A have break-even periods of less than 5 years. Second, once the interest rate rises to 10% or more there are very few instances when the IRA does not break even within 10 years. Thus, to make the IRA worthwhile, the one thing that investors must ensure is a high interest rate.[e] The time of withdrawal is the third characteristic that short break-even periods have in common. The later the withdrawal occurs, the faster the IRA is likely to break even.

Second, under what circumstances should investors stay away from the IRA? Since short break-even periods have at least two important common denominators, small withdrawals and high interest rates, long break-even periods arise in situations when both are absent. For instance, if an investor in the 15% tax bracket withdraws 75% of his accumulated IRA which has been earning interest only at 4%, then the break-even period could be as high as 47.4 years after the withdrawal depending upon when the amount is withdrawn. It looks like all the long break-even periods appear in the area where the interest rate is low and a large proportion of the IRA is withdrawn. If an investor earns a low interest rate due to risk aversion and if that investor is likely to withdraw a substantial proportion of the IRA in an emergency due to lack of resources, then that investor should stay away from an IRA account because under these circumstances it usually takes several years after withdrawal for the IRA to break even.

Finally, how do the break-even periods reported in Table 2 compare with the ones reported previously in the practitioner oriented and in the academic literature? Bernstein (1993), who may be representative of the former group, states:

> ...the breakeven point is generally around 7 years. That is, you are generally better off with an IRA if the money is left in for about 7 years or more and then withdrawn prematurely (with 10% penalty) than you would have been with a comparable investment outside the IRA. (p. 226)

It is not known what assumptions were made by Bernstein to conclude that 7 years is the typical break-even period for IRAs. However, Bernstein's conclusion is consistent with Collins' (1980) who states "...the break even time is around seven years for most marginal tax rates" (p. 132). Collins assumes an interest rate of 7% and tax rates that constant throughout the investor's life. Thus, it appears that both in the practitioner oriented literature and the academic literature 7 years is considered to be representative of the average break-even period. On the other hand, the average of the values appearing in Table 2 is 3.40 years.[f] Therefore, the results presented here suggest that the break-even period on a typical IRA that is withdrawn early and penalized is shorter than what has been reported in the literature previously.

Conclusions

In this paper, an expression that gives the break-even period for a partially withdrawn IRA is derived when the partial withdrawal is penalized, but the remaining amount is not. While other papers have addressed the issue of fully withdrawn IRAs that are penalized, the approach in this paper is more general because it allows for partial withdrawals and it recognizes that once the withdrawal occurs, investors still have a choice as to how much longer they want to remain invested in the IRA.

The comparative-statics of the break-even period are also presented. Although in general the signs of the partial derivatives of the break-even period are indeterminate, after placing certain restrictions on the parameters under which the partials are evaluated, some interesting and important conclusions can be made. These comparative-static results suggest that the break-even period falls as the interest rate rises, or the investor's marginal tax rate increases, or when the withdrawal is delayed, and the break-even period rises with the proportion withdrawn.

Finally, results are presented on the break-even period for a variety of different tax rates, proportion withdrawn, interest rates, and time of withdrawal. The results suggest that as long as the IRA earns interest at 10% or more or a small proportion is withdrawn (25% or less), it is practically guaranteed that the IRA will break even within a short time after withdrawal.

If an investor is likely to withdraw a large proportion of his IRA due to the unavailability of other means, *and* if that investor is able to invest only at a low interest rate, the results suggest that he is better off with ordinary investments because the break-even period becomes relatively large. Since the latter would occur only under a low tolerance for risk, the IRA is an attractive alternative to ordinary investments even in the presence of withdrawal uncertainty. This is further supported by the average break-even period of 3.40 years for the data appearing in this paper which is less than half of the 7 years that has been previously reported in the literature.

Notes

a. *For a contribution to be considered deductible, one of two conditions must be met: (1) neither the taxpayer nor his spouse must be covered by a retirement plan or (2) the adjusted gross income must fall below a certain level. Even if the taxpayer is not eligible to make deductible contributions, he may be allowed to make non-deductible contributions. For a succinct analysis of conditions under which deductible contributions may be allowed, and for more information on nondeductible contributions see Chapter 2 in The Institute of Financial Education (1993).*

b.

$$\frac{\partial n^*}{\partial t} = \frac{\dfrac{\alpha e^{rjt} rt(1-p-t)[1-(1-\alpha)(1-t)rj]}{(1-\alpha)(1-t)[(1-t)-\alpha(1-p-t)e^{rjt}]} - \ln\left(\dfrac{(1-t)-\alpha(1-p-t)e^{rjt}}{(1-\alpha)(1-t)}\right)}{(rt)^2},$$

$$\frac{\partial n^*}{\partial \alpha} = \frac{(1-t)-(1-p-t)e^{rjt}}{rt(1-\alpha)[(1-t)-\alpha(1-p-t)e^{rjt}]},$$

$$\frac{\partial n^*}{\partial r} = \frac{-t\ln\left(\dfrac{[(1-t)-\alpha(1-p-t)e^{rjt}]}{(1-\alpha)(1-t)}\right) + \dfrac{\alpha(1-p-t)rjt^2 e^{rjt}}{(1-t)-\alpha(1-p-t)e^{rjt}}}{(rt)^2},$$

and $$\frac{\partial n^*}{\partial j} = \frac{-\alpha(1-p-t)e^{rjt}}{(1-t)-\alpha(1-p-t)e^{rjt}}.$$

c. *There is no way to demonstrate these results analytically. The contentions on the signs of the partial derivatives are based on numerical results which are not reproduced here, but available separately from the author.*

d. *Since IRA contributions are tax-deductible at the state level in some states, the tax rate that is used in equation A8 should be higher for individuals residing in those states. To retain generality, state taxes are ignored.*

e. *Even though interest rates are listed as being the second most important characteristic that short break-even periods share, they are of paramount importance in deciding whether to invest in a IRA or not because α is not under investors' control while interest rates are—by appropriately choosing the risk characteristics of their investments.*

f. *Clearly, all scenarios presented in the table are not equally likely. In fact, since no evidence is available on when IRAs are withdrawn, it is not known which scenarios are more likely. The mean that is reported here is purely for the purpose of comparison.*

g. *With the possibility of income variability, an individual's tax rate could change from year-to-year. One way of making the model presented here more realistic is to allow for different tax rates every year. However, it would be impossible to make any calculations on the break-even period since there would be an infinite number of different tax rate sequences. One possibility is to allow for the tax rate during the working years to be different from the tax rate at withdrawal. This is the approach adopted in Clute and Reichenstein (1989) and in Collins (1980). Another possibility is to assume that the tax rate is constant throughout the investor's life. Bogan and Bogan (1982) and Smith (1984) take this approach. The latter approach is taken in this paper because uncertainty of withdrawals plays a key role in the motivation for this paper. Such uncertainty would suggest that the tax rate at withdrawal is the same as the tax rate during the working years because withdrawals occur during working years in this paper.*

Appendix

The model that gives the break-even period for a partially withdrawn IRA is presented here. It is assumed that the investor's marginal tax rate remains constant throughout the investor's life.[g] If an investor contributes \$A to an IRA that earns interest at a rate of r% compounded continuously, then the IRA will grow to

$$X^j_{IRA} = Ae^{rj} \qquad \textbf{(A1)}$$

where X^j_{IRA} is the accumulated amount in the IRA at time j. If E is the investor's emergency cash need at time j, then $\alpha \in (0,1)$ is the proportion of X^j_{IRA} that has to be withdrawn to meet the need, so

$$\alpha = \frac{E}{Ae^{rj}(1-p-t)} \qquad \textbf{(A2)}$$

where p is the IRS penalty for early withdrawal, and t is the investor's marginal tax rate. The remaining amount continues to earn interest tax-free at interest rate r till maturity which, by assumption, always occurs at or after the investor reaches age 59½. Since the final withdrawal is not subject to penalty, by period n the remaining amount will have grown to

$$X^n_{IRA} = (1-\alpha)Ae^{rn}(1-t) \qquad \textbf{(A3)}$$

on an after-tax basis. This amount must be compared to the ordinary investment to determine what the IRA's break-even period is.

Since contributions to ordinary investments are non-deductible, an IRA contribution of \$A will be equivalent to an ordinary investment of (1-t)\$A. At time j of the emergency withdrawal, the ordinary investment will have grown to

$$X^j_{ORD} = (1-t)Ae^{r(1-t)j} \qquad \textbf{(A4)}$$

where X^j_{ORD} is the amount accumulated in the ordinary account at the continuously compounded after-tax interest rate of r(1-t)%. After withdrawing E from this account, the remaining amount will grow to

$$X^n_{ORD} = (X^j_{ORD}-E)e^{r(1-t)(n-j)} \qquad \textbf{(A5)}$$

Substituting for X^j_{ORD} from (4) and for E from (2), and by rearranging the terms, X^n_{ORD} can be written as

$$X^n_{ORD} = A\left[(1-t)-\alpha(1-p-t)e^{rjt}\right]e^{r(1-t)n} \qquad \textbf{(A6)}$$

The IRA breaks even at n^* such that $X^{n^*}_{IRA}=X^{n^*}_{ORD}$. To find n^*, set equation 3 equal to equation 6 and solve for $n=n^*$ which gives

$$\frac{e^{rn}}{e^{r(1-t)n}} = \frac{\left[(1-t)-\alpha(1-p-t)e^{rjt}\right]}{(1-\alpha)(1-t)} \qquad \textbf{(A7)}$$

Simplifying the left hand side and taking the natural log of both sides yields

$$n^* = \frac{\ln\left(\dfrac{\left[(1-t)-\alpha(1-p-t)e^{rjt}\right]}{(1-\alpha)(1-t)}\right)}{rt} \qquad \textbf{(A8)}$$

Equation A8 gives the break-even period n^* for a given tax-rate, interest rate, time of early withdrawal, proportion withdrawn, and penalty.

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