Optimal Portfolios for Retirement

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Using Ibbotson's Historical Data on Securities prices, simulations were run to determine the optimal portfolio for an individual who simultaneously consumes and invests in order to finance current consumption. The simulations assumed that households consume according to the Life Cycle Income Hypothesis: each year, they consumed in proportion to their life expectancy. The results yielded several counterintuitive results, the most prominent being that under all assumptions, retiree portfolios should be less aggressive than those using a simple 1 year time horizon.

Key Words: asset allocation, investment, life cycle income hypothesis, retirement

Background

The objective of this paper was to determine optimal investment portfolios by evaluating, through simulation, the consumption patterns of a hypothetical consumer who each year withdraws from his or her portfolio some amount that is determined in some way or another by the return on that portfolio. In particular, we were concerned with the retiree who simultaneously consumes and invests in order to finance current consumption.

The investments were restricted to six choices. They were well-diversified portfolios of small-capitalization stocks, large-capitalization stocks, corporate bonds, and long-, short- and intermediate-term government bonds, respectively, as defined by Ibbotson (2006), which gave the historical returns on these six types of assets from 1926-2005. Table 1 gives the mean and standard deviations of returns for these investments over that time period. Note that if individuals desire high rates of return with low standard deviations in those returns, then none of these four investment types dominate any of the others; the higher the expected rate of return, the more the volatility in returns.

Table 2 illustrates the asset allocation decisions of individuals at traditional retirement age. It shows the proportion of investment assets (that are not in any type of pension) in equities (stocks) or equity-based mutual funds for heads of households over 50. Although some key elements of financial assets are left out of this table, it implies that older individuals invest less of their investment assets in the stock market. We have also seen the proportion of assets held in stocks increase from 1995 to 2004, either because investors' demand for higher-return, higher-volatility assets has increased, or because investors have imperfectly rebalanced their portfolios from the high stock market returns relative to bonds from the late nineties.

The figures from Table 2 correspond somewhat to financial industry recommendations regarding asset allocation (e.g., subtract current age from 100 and that is the percentage of savings one should have in the stock market). With respect to analytical approaches to this problem, previous literature has dealt primarily with the question of optimal investments that depend on *investment horizon.* Under this scenario, expected utility analysis was used to derive optimal investments, depending upon the level of risk tolerance that was assumed in the utility function, and the amount of time before the investment in question was to be consumed (i.e., time horizon). Hanna and Chen (1997) used this approach using the Ibbotson data and the following assumption about utility…

$$U(W) = \frac{W^{1-x}}{1-x}; x < 1 \quad (1)$$

…where W is total wealth and x is the level of risk aversion. Hanna and Chen asked the following question. Given someone's risk aversion level, x and time horizon n, which portfolio would have generated the highest expected utility if the consumer had started his or her investment program

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Table 1. Historical Returns: 1926-2005

	Rate of return	
	M	*SD*
Small company stocks	.175	.329
Large company stocks	.123	.202
Long-term corporate bonds	.062	.085
Long-term government bonds	.058	.092
Intermediate-term government bonds	.054	.057
U.S. Treasury bills	.037	.032
Inflation	.031	.043

in each year from 1926 until (1995-n) and consumed the proceeds n years later? The candidates for portfolios were all those with integer percentage amounts from 0 to 100 in each of the Ibbotson categories, as long as the sum of percentages in all categories sums to 100. Therefore, Hanna and Chen's model of consumption was one where the investor only cares about how much consumption one's investment will yield exactly n years later. Since it would be impractical to assume that the individual in question would consume only those investment proceeds, Hanna and Chen ran simulations for two cases: one where the investment portfolio is only 10% of total wealth (of which the other 90% also generates a reasonable consumption return each period) and one where the investment portfolio is 50% of total wealth. The 10% simulations, therefore, correspond to the person whose investment assets make up very little of their consumption (so, presumably, they can invest them more aggressively, controlling for risk aversion and time horizon) and the 50% simulations correspond to investors with more substantial holdings.

Table 3 is a reproduction of Hanna and Chen's results for investors with a 1 year time horizon. Hanna and Chen stated, "A 1 year (investment horizon) is appropriate for investors whose current consumption depends upon the portfolio" (Hanna & Chen, 1997, p. 20). Intuitively, we would expect this to be an under-estimation of the actual time horizon, since most retirees will be investing for longer than 1 year. On the other hand, the actual length of time retirees are investing (perhaps the retiree's life expectancy) would be inappropriate, since the retiree would also be interested in the period-specific returns on his or her portfolio and not just some ending value. For these two reasons, we might expect retirees to invest in portfolios more aggressive than Hanna and Chen's 1 year portfolios

Table 2. Proportion of Non-Pension, Non-Annualized Investment Assets in Stocks

Age of household head	1995	2004
50-59 years	.160	.245
60-69 years	.132	.227
70-79 years	.104	.173
80 years and over	.097	.172

and yet not as aggressive as investors with time horizons as long as the individual's life expectancy.

Substantial literature exists that attempts to ascertain optimal retirement allocation strategies by determining which allocations can sustain the highest withdrawal rates with the lowest probability of "ruin" – running out of money before one's life. Most of this literature came to the conclusion that portfolios substantially more aggressive than Hanna and Chen's 1 year portfolios are preferred in retirement (Bengen, 1997; Tezel, 2004). Cooley, Hubbard, and Walz (1999), for example, determined the percentage of times that different portfolios end with positive asset values for different fixed levels of per-period withdrawal (as a percentage of the initial portfolio value) and different time horizons. They found that portfolios as high as 75% in the stock market should sustain reasonable withdrawal rates (4%-8%) well into retirement (as long as 40 years).

The limitation of this type of approach is that the portfolios in question have no guarantee of financial sustainability, only a high probability. These models did not guarantee that a consumer would not run out of money using these fixed-withdrawal rules, however, improbably it was in the past. Furthermore, these simulations presumed that an individual's consumption is not at all sensitive to the portfolio's rate of return. This is consistent with the Life Cycle Income Hypothesis with a fixed assumption about the interest rate that is never revised (Ando & Modigliani, 1963). It is not consistent with actual retiree behavior, however, since consumption in retirement actually depends, to some extent, on retiree returns on investments. Subsequent studies have attempted to adjust for this concern through "adjusting withdrawal rates" that allow rates to rise or fall based on portfolio performance (Bengen, 2001; Pye, 2001; Stout & Mitchell, 2006). Still, while these models can show that they never would have failed in the past, they can offer no such guarantee for the future.

Table 3. Hanna and Chen's 1 Year Portfolios

	Risk aversion level		
	Low (x = 2)	Middle (x = 6)	High (x = 10)
Portfolio is 10% of total wealth			
% Invested in small stocks	100	100	100
% Invested in large stocks	0	0	0
% Invested in corporate bonds	0	0	0
% Invested in government bonds	0	0	0
Portfolio is 50% of total wealth			
% Invested in small stocks	100	18	10
% Invested in large stocks	0	32	21
% Invested in corporate bonds	0	0	0
% Invested in government bonds	0	50	69

Model

Instead of assuming that individuals consume fixed predetermined amounts each year, this paper made the rather simplifying assumption that consumption in retirement corresponds to Ando and Modigliani's (1963) assumptions; namely, that in retirement, households consume the quotient of their asset pool and their life expectancy. This is Ando and Modigliani's original interpretation of the model; it should not be confused with more recent interpretations, such as constant consumption over time. This assumption has several desirable features. First, it is sustainable; households never run out of assets using this strategy. Second, under reasonable interest rates, consumption will (eventually) decline over time, which is consistent with actual behavior (see, for example, Hitschler, 1993), although declining consumption in retirement may be due to poor financial planning in the first place (referred to commonly as the "retirement savings puzzle" – see Banks et al., 1998). Third, it incorporates some of the consumption-smoothing that makes the Life Cycle Income Hypothesis so attractive, both as a way of explaining behavior and as a practical recommendation (most financial planners, if textbooks on the topic are to be believed, make at least some consumption-smoothing assumptions when helping clients plan for retirement). Last, it does not go so far as to render the return one receives year to year as irrelevant in the consumption decision; households that earn higher returns will adjust their consumption upward, and vice-versa.

Table 4 shows hypothetical consumption rates under this consumption regime from a pool of $100,000 for an individual age 60–90, using assumptions about life expectancy from the National Center for Health Statistics (2006) and two different assumptions about returns. Using a relatively low real rate of return (2%), consumption from the asset pool starts at approximately $4,500 at age 60, increases slightly until around age 70, whereby it decreases. The asset pool drops throughout this period from the start, consistent with how most researchers and practitioners interpret the Life Cycle Income Hypothesis. If the household was fortunate enough to earn a consistently high real rate of 8%, consumption would increase significantly from the start but would eventually decrease around age 73, and assets would begin to be spent down by age 76. Note that if asset returns are high enough, the fact that asset spend down does not happen until later in retirement and retirement consumption increases *is* consistent with the Life Cycle Income Hypothesis. The credibility of the following results depend critically on how reasonable it is to assume that consumption in retirement follows the rules implied in Table 4.

Wealth is made up of two components – the portfolio, which earns real rates of return based on simulation outlined below, and non-portfolio wealth (e.g., social security contingent claims, home equity, pension claims, etc.), which is assumed to earn a 2% real return. Wealth at any given period is equal to wealth in the previous period, minus consumption in the previous period (wealth divided by life expectancy), plus the return in the previous period. Wealth at the beginning of the period is arbitrary, since the utility models used below are constant with respect to initial wealth. This means, for example, that the optimal portfolio for a $1.00 initial portfolio is the same as one for that of $1 million. The simulations are influenced, however, by what proportion of one's wealth is allocated to the portfolio and how much is allocated to non-portfolio

Table 4. Hypothetical Consumption Under the Life Cycle Income Hypothesis

Age	Life expectancy	2% Real return		8% Real return	
		Spending	Final assets	Spending	Final assets
60	22.2	4,504.50	97,405.41	4,504.50	103,135.14
61	21.5	4,530.48	94,732.42	4,796.98	106,205.20
62	20.7	4,576.45	91,959.09	5,130.69	109,160.48
63	19.9	4,621.06	89,084.79	5,485.45	111,969.03
64	19.2	4,639.83	86,133.86	5,831.72	114,628.30
65	18.4	4,681.19	83,081.73	6,229.80	117,070.38
66	17.7	4,693.88	79,955.60	6,614.15	119,292.73
67	17.0	4,703.27	76,757.38	7,017.22	121,257.55
68	16.3	4,709.04	73,489.30	7,439.11	122,923.91
69	15.6	4,710.85	70,154.02	7,879.74	124,247.71
70	14.9	4,708.32	66,754.61	8,338.77	125,181.65
71	14.4	4,635.74	63,361.25	8,693.17	125,807.56
72	13.6	4,658.92	59,876.38	9,250.56	125,881.56
73	13.0	4,605.88	56,375.92	9,683.20	125,494.24
74	12.4	4,546.44	52,866.06	10,120.50	124,603.63
75	11.8	4,480.17	49,353.60	10,559.63	123,167.52
76	11.2	4,406.57	45,845.97	10,997.10	121,144.05
77	10.6	4,325.09	42,351.30	11,428.68	118,492.60
78	10.1	4,193.20	38,921.26	11,731.94	115,301.51
79	9.5	4,096.97	35,520.77	12,137.00	111,417.67
80	9.0	3,946.75	32,205.50	12,379.74	106,960.96
81	8.5	3,788.88	28,984.95	12,583.64	101,927.51
82	8.1	3,578.39	25,914.69	12,583.64	96,491.37
83	7.6	3,409.83	22,954.96	12,696.23	90,498.75
84	7.2	3,188.19	20,162.11	12,569.27	84,163.84
85	6.8	2,965.02	17,541.03	12,377.04	77,529.75
86	6.4	2,740.79	15,096.25	12,114.02	70,648.98
87	6.0	2,516.04	12,831.81	11,774.83	63,584.08
88	5.6	2,291.40	10,751.23	11,354.30	56,408.17
89	5.3	2,028.53	8,897.15	10,643.05	49,426.33
90	5.0	1,779.43	7,260.07	9,885.27	42,704.35

wealth. Here, the same values are assigned for this proportion as in Hanna and Chen: 10% and 50%. While there is some evidence that the average household does not hold investment assets in such high proportions of their wealth (Gutter, 2000), they were used here to be consistent with Hanna and Chen.

This new model required that we expand Hanna and Chen's model to include consumption streams, not just consumption levels at one point in time. This expected utility model of consumption in retirement was done using essentially the same Hanna and Chen formula…

$$V = \sum_{i=0}^{l} \frac{c_i^{\,1-x}}{(1-x)}(1+\rho)^{-i} : x > 1 \qquad (2)$$

…where V is the consumption of a particular stream of consumption, c_i is the consumption at age i, x is a parameter for regulating the marginal utility of consumption, l_i is the life expectancy of the household, and ρ is a discount factor.

As in Hanna and Chen (1997), the results here were run using non-parametric Monte Carlo simulation (Ibbotson, 2006). Instead of using parametric methods (assuming that different asset classes correspond to parameterized distributions over time), we assumed here that each overlapping n-year epoch from 1926-2006 are equally likely to occur over the next n years, for each asset class, where n is either 20 or 30 years, for every conceivable integer percentage combination of asset. The results of each simulation are adjusted by the risk aversion factor, x…

$$U = \sum_{i=1}^{81-n} \frac{V_i^{\,1-x}}{1-x} ; x > 1 \qquad (3)$$

…where U is expected utility, V is the utility of one contingent consumption stream, as in (2), n is the length of retirement, and x is the risk aversion factor. Since Ibbotson had 81 years of data, there are (81-n) unique epochs of n-year asset experiences. The variable x now has double duty; it is the degree to which consumption is substitutable across periods and a measure of relative risk aversion. This is an additional assumption; households may be willing to tolerate volatility with the overall level of consumption across states of the world but not to consumption between periods, or vice-versa. The model does not distinguish between the two concepts. A simulation where the consumption stream is low gets more weight than when it is high (the higher x, the more weight is given to lower consumption streams).

The optimal portfolio, as in Hanna and Chen (1997), is that portfolio with integer percentage amounts of each of the six classes of assets, out of all candidate portfolios (those where the percentages sum to 100), that gives the consumption streams with the highest expected utility, given: a pre-specified length of retirement (the results here are for a retiree with 20 and 30 years of retirement, representing an older and younger retiree, respectively); the risk aversion/intertemporal substitution of consumption factor x; a discount rate ρ; and, as in Hanna and Chen, the proportion of one's total wealth made up by the portfolio. A discount rate was used to acknowledge a relative preference for present consumption; an affinity for earlier consumption might be due to a more likely chance of surviving, for example, or it might just represent general myopia or preferences for the present unrelated to survival probabilities. It is also important to note that under this model of consumption different asset allocations would ultimately yield different bequests (i.e., assets left over at the end of the simulations) so that for these simulations to be considered optimal, an additional assumption that bequests are irrelevant to the household (or at least that the differences between the bequests of the different asset allocation alternatives are irrelevant) must be made.

Results

Table 5 presents the optimal portfolios using the model described above, when the portfolio represents 10% of the investor's total wealth. It is compared to Hanna and Chen's 1 year portfolio (recall that this is Hanna and Chen's recommendation for the retiree investor). While all six classes of assets from Ibbotson were used in the simulations, only four are reported here, because two of the asset classes had an allocation of zero in all of the simulations (intermediate-term bonds and U.S. Treasury bills). The optimal portfolios calculated using the Life Cycle Income Hypothesis were actually more conservative than the Hanna and Chen 1 year portfolios. Depending on risk aversion, the discount rate, and the length of the simulation (20 or 30 years), optimal portfolios calculated here have between 35% - 92% invested in the stock market, while all of Hanna and Chen's 1 year portfolios are 100% invested in small-capitalization stocks. Not surprisingly, when a high risk aversion was assumed, the optimal portfolio was less aggressive (more bonds, less stocks). When a higher discount rate was used (meaning that the

Table 5. Optimal Allocation of Investment Portfolios (Assuming Portfolio Is 10% of Wealth)

	Relative risk aversion		
	Low (x = 2)	Middle (x = 6)	High (x = 10)
Hanna and Chen 1 year portfolios			
Small stocks	100	100	100
Large stocks	0	0	0
Corporate bonds	0	0	0
Long-term US bonds	0	0	0
Retiree with life expectancy of 20 years (discount rate = .02)			
Small stocks	76	52	33
Large stocks	10	4	5
Corporate bonds	0	0	0
Long-term US bonds	14	44	62
Retiree with life expectancy of 20 years (discount rate = .33)			
Small stocks	82	53	33
Large stocks	10	4	5
Corporate bonds	0	0	0
Long-term US bonds	8	43	62
Retiree with life expectancy of 30 years (discount rate = .02)			
Small stocks	69	42	25
Large stocks	10	10	10
Corporate bonds	0	0	0
Long-term US bonds	21	48	65
Retiree with life expectancy of 30 years (discount rate = .33)			
Small stocks	74	43	22
Large stocks	10	10	15
Corporate bonds	6	0	0
Long-term US bonds	10	47	63

beginning of the retirement period was relatively more meaningful in the utility calculation than the end of the period), the optimal portfolios were more slightly aggressive (more stocks, less bonds). This might seem counterintuitive, because discount rates are often equated with time horizon (i.e., that people with higher discount rates have longer time horizons). Here, it means that the returns that happen in the beginning of the period are going to be more important than those at the end, which could be the result of a historical idiosyncrasy of the data. Nevertheless, the effect was extremely slight, since a 33% discount rate was quite high (myopic) compared to the more reasonable assumption of 2%. The main result here was that discount rate assumptions did not seem to affect optimal portfolios.

The fact that the assumption of a longer life expectancy had *ceteris paribus* led to less aggressive portfolios is also counter-intuitive. One would expect that the longer a retiree had to live, the more aggressive their portfolio would be, not vice-versa. It is the mechanism by which the stream of consumption is evaluated in the utility function that allows for this result. A 30 year old retired household must plan for 30 years of consumption, and a more aggressive portfolio has the possibility of generating much more variant consumption, using the Life Cycle Income Hypothesis model, than if there were only 20 years. This creates an implicit preference in the model for slightly less aggressive portfolios for households with longer time horizons.

Table 6 gives similar results as Table 5 for households investing 50% of their wealth instead of 10%. In general, the resulting portfolios were less aggressive than those in Table 1 (just like the Hanna and Chen portfolios). The

Table 6. Optimal Allocation of Investment Portfolios (Assuming Portfolio Is 50% of Wealth)

	Relative risk aversion		
	Low (x = 2)	Middle (x = 6)	High (x = 10)
Hanna and Chen 1 year portfolios			
Small stocks	100	18	10
Large stocks	0	32	21
Corporate bonds	0	0	0
Long-term US bonds	0	50	69
Retiree with life expectancy of 20 years (discount rate = .02)			
Small stocks	63	26	16
Large stocks	6	2	1
Corporate bonds	0	0	0
Long-term US bonds	31	72	83
Retiree with life expectancy of 20 years (discount rate = .33)			
Small stocks	65	26	17
Large stocks	5	1	0
Corporate bonds	0	0	0
Long-term US bonds	30	73	83
Retiree with life expectancy of 30 years (discount rate = .02)			
Small stocks	54	16	10
Large stocks	10	12	7
Corporate bonds	0	0	0
Long-term US bonds	36	72	83
Retiree with life expectancy of 30 years (discount rate = .33)			
Small stocks	55	16	10
Large stocks	10	12	7
Corporate bonds	0	0	0
Long-term US bonds	35	7	83

results from the simulations calculated here were still substantially less aggressive than the Hanna and Chen 1 year portfolios, and all of the other results gleaned from Table 5 apply to Table 6 as well. When the assumption about risk aversion was increased, the portfolio was less aggressive. Higher discount rates lead to very slightly more aggressive portfolios, and retirees with a longer time horizon had less aggressive portfolios.

Conclusions

When undertaking this research, the author certainly expected to find optimal portfolios that were more aggressive (in terms of percent invested in stocks) than the Hanna and Chen 1 year portfolios. However, the intuitive notion that optimal portfolios for retired persons should be somewhere between life expectancy and one year does not appear to be the case, assuming that consumption strictly follows the Life Cycle Income Hypothesis. Theoretically speaking, the simulations yielded such conservative asset allocation because under this consumption regime, retirees not only have to worry about their asset level but also have to worry about consumption fluctuation. The model does not allow retirees to sock away all the windfall gains from high returns for a rainy day; you spend some of it and you save some, *a la* Ando and Modigliani, and this can result in rather variant consumption over the life course, unless you invest conservatively. The way this model works, fluctuation impacts consumption substantially more than in Hanna and Chen's world. For example, with the Hanna and Chen 10 year portfolio, it is assumed that households care nothing about within-period fluctuation, only the final asset level after 10 years (a reasonable, though not proven assumption). Here, the household actually does care about within-retirement asset fluctuation because the model forces them to consume based on that fluctuation. It is not clear that this rather rigid assumption is reasonable, and it begs for further research on how to model consumption in retirement, not only for optimal asset allocation research but retirement research in general.

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